MATH 142 Midterm Exam #1

Sept 30, 2005 11:30AM, Prof Johnson

NAME:_____

- **No calculators are allowed on this exam.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about u-substitutions, and use correct mathematical grammar in the presentation of your solution.**

Problem	Points	Score
1	20	
2	30	
3	15	
4	15	
5	20	
total	100	

$$\sum_{i=1}^{n} a = a \cdot n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

1. Definition of the Integral. Recall the definition of the definite integral for a continuous function $f(x)$ on the interval $[a, b]$.

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Calculate the integral below using the definition of the integral.

$$\int_0^4 3 - x\ dx$$

(a) First, find the following quantities:

$\Delta x =$ _____ $x_i =$ _____ $f(x_i) =$ _____

(b) Next, using the quantities above and the summation formulas on the front page of the exam, simplify $\sum_{i=1}^{n} f(x_i)\Delta x$ into an expression without the summation notation.

(c) Last, evaluate the limit, $\lim_{n\to\infty}(\sum_{i=1}^{n} f(x_i)\Delta x)$.

Note: you can *check* your answer by using the Fundamental Theorem of Calculus.

2. Integrals. Evaluate the following definite and indefinite integrals.

(a) Let $f(x) = \begin{cases} 2 & \text{if} \quad x \leq -1 \\ -5 & \text{if} \quad -1 < x \leq 4 \\ 0 & \text{if} \quad x > 4 \end{cases}$

Calculate $\displaystyle\int_{-3}^{6} f(x)\, dx.$

(b) $\displaystyle\int_1^4 \frac{3}{t^2} - \frac{3\sqrt{t} + 4t^3}{t}\, dt$

(c) $\displaystyle\int x\sqrt{2x+1}\, dx$

3. The velocity of an object moving along a straight line is given by $v(t) = \sqrt[5]{e^t}$ meters/minute, where t is time in minutes.

   Find the change in position of the object over the time interval $2 \leq t \leq 6$ minutes.

4. Find the area between the curve $y = \cos(\frac{x}{2})$ and the x-axis over the interval $0 \leq x \leq 3\pi$.

5. Volume. Note: On this problem, you can earn partial credit for parts (b)-(e) by sketching an arbitrary slice of the volume in the space provided in the left margin.

Consider the region, R which is bounded between the curves

$y = 4x - x^2$ $\qquad$ $y = x$

(a) Sketch the region and label the points of intersection.

(b) Write an integral for the volume of the solid formed by rotating the region R about the y-axis. DO NOT evaluate the integral.

(c) Write an integral for the volume of the solid formed by rotating the region R about the line $y = -1$. DO NOT evaluate the integral.

(d) Now consider the solid whose base is the region R and whose cross-sections above the xy-plane and perpendicular to the x-axis (i.e. slices parallel to the y-axis) are squares. Write an integral for the volume of this solid, but DO NOT evaluate the integral.