MATH 142 Midterm Exam #2

November 2, 2005 1:50PM

NAME:_____

- **No calculators are allowed on this exam.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about the u-substitutions or integration by parts choice(s) that you make.**

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$	$\cos^2 x = \frac{1}{2}(1+\cos(2x))$	$\sin(2x) = 2\sin(x)\cos(x)$

Expression	Substitution	
$\sqrt{a^2-x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2+x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2-a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

Problem	Points	Score
1	51	
2	16	
3	12	
4	6	
5	15	
total	100	

1. Integrals [17 points each]

DO EXACTLY THREE OF THE FOUR INTEGRALS BELOW!! Do not do all four problems. Make it clear which problem you are not attempting.

Evaluate the integrals. Show all work and include all information about substitutions and integration by parts choices, and restrictions on angles for substitutions etc. Simplify all trig expressions.

(a) $\displaystyle\int \frac{2}{2x^2 + x - 1}\, dx$

(b) $\displaystyle\int_1^4 x^6 \ln(x)\, dx$

(c) $\displaystyle\int_0^{\pi/3} \cos^7(x)\tan^3(x)\,dx$

(d) $\displaystyle\int \frac{1}{(9+16x^2)^{3/2}}\,dx$

[16 points]

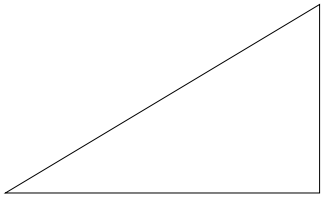
2. The cosine function is not one-to-one, so its domain is restricted to $[0, \pi]$ in order to define the inverse cosine function.

$\cos^{-1}(x) = y$	means	$\cos(y) = x$ and, $0 \leq y \leq \pi$

Follow the steps below to find the derivative of the inverse cosine function $\dfrac{d}{dx}(\cos^{-1}(x))$.
Note: You will not earn full credit for writing down the formula for the derivative of the inverse cosine function. You should explain where the formula comes from by following the steps below.

(a) Use the definition of the inverse cosine function, $\cos(y) = x$, to find a formula for $\frac{dy}{dx}$.

(b) Label all three sides of the right triangle to reflect the fact that $\cos(y) = x = \frac{x}{1}$.



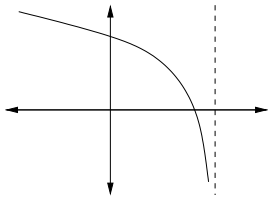
(c) Use the triangle above to find the formula for $\frac{dy}{dx}$ in terms of x.

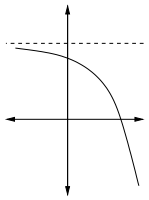
$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{dy}{dx} =$$

(d) Calculate $\frac{d}{dx}\left(\cos^{-1}(4x^3 + 2x)\right)$

3. <u>True or False.</u> In the blank provided, write TRUE if the statement is always true or FALSE if the statement is sometimes false. [3 points each]

_____ If the graph of $y = f(x)$ is given below on the left, the the graph of $f^{-1}(x)$ will look like the graph given below on the right.





_____ The integration formula for integration by parts comes from the chain rule derivative formula.

_____ The quantity $\log_5\left(\frac{1}{5^2}\right)$ can be simplified to $-\frac{1}{2}$.

_____ The solution to the equation $4^x = 5$ is $\log_5(4)$.

4. Evaluate the following quantities: [6 points]

$\sin^{-1}(\sin(\pi)) =$

$\cos^{-1}(0) =$

5. Find the inverse function of $f(x) = \dfrac{5x+2}{x-3}$. [15 points]