MATH 142

Final Exam

December 12, 2006

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

- **The first part of the final can replace your lowest midterm score, but it will also count towards your score on the final. If you skip it, you will get at most 100 points out of 200.**
- **No calculators are allowed on this exam.**
- **Please put your final answers in the spaces provided.**
- **When integrating, put down all information you are using, such as substitutions or integration by parts.**
- **You do not need to simplify expressions such as** $\frac{3}{4^2} + 26(3)^4 - \frac{\pi}{2}$**, but you do need to evaluate expressions such as** $\sin(\pi/4)$**.**

Part A		
QUESTION	VALUE	SCORE
1	10	
2	50	
3	10	
4	20	
5	10	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
6	20	
7	15	
8	10	
9	15	
10	20	
11	10	
12	10	
TOTAL	100	

Part A

1. (10 pts) The definition of the definite integral for a continuous function $f(x)$ on the interval $[a, b]$ using righthand endpoints for the Riemann sum is given below.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Calculate the integral below using the definition of the integral.

$$\int_0^2 1 - 2x^2\,dx$$

(a) First, find the following quantities:

$\Delta x =$ ____________ $x_i =$ ____________ $f(x_i) =$ ________________________

(b) Next, using the quantities above and the summation formulas on the front page of the exam, simplify $\sum_{i=1}^{n} f(x_i)\Delta x$ into an expression without the summation notation.

(c) Last, evaluate the limit, $\lim_{n\to\infty}(\sum_{i=1}^{n} f(x_i)\Delta x)$.

Note: you can *check* your answer by using the Fundamental Theorem of Calculus.

2. (50 pts) Evaluate **FOUR** of the following FIVE integrals. Do not do all five integrals. If you do all five integrals I will grade the first four.

(i) $\displaystyle\int \cos^3(x)\sin^6(x)\ dx$

ANSWER: ______________________

(ii) $\displaystyle\int_0^{\sqrt{\pi/2}} x^3\cos(x^2)\ dx$

ANSWER: ______________________

(iii) $\displaystyle\int \frac{e^x}{e^{2x} + e^x}\ dx$

ANSWER: ______________________________

(iv) $\displaystyle\int_{-1}^{4} \frac{2}{(x-1)^2}\ dx$

ANSWER: ______________________________

(v) $\displaystyle\int \frac{1}{x^2\sqrt{9x^2+16}}\,dx$

ANSWER: ______________________________

3. (10 pts) Show that the sphere of radius 1 has volume $\frac{4}{3}\pi$ by following the steps below.

a) Write the equation which describes the set of all points (x, y) which lie on the circle of radius 1 centered at the origin. Draw this circle in the xy-plane below.

b) Set up an integral which calculates the volume of the solid obtained by rotating the area inside the top half-circle about the x-axis.

c) Evaluate the integral from part b).

4. (20 pts) Consider the region R bounded by the curves

$$y = e^x, \qquad x = 1, \qquad x = 2 \qquad \text{and} \qquad y = 0.$$

(a) Sketch the curves given above, label the points of intersection, and shade the region bounded by the curves.

(b) Set up but DO NOT EVALUATE an integral which calculates the *volume* of the solid obtained by rotating the region about the line $y = -1$.

(c) Set up but DO NOT EVALUATE an integral which calculates the *volume* of the solid obtained by rotating the region about the line $x = -4$.

5. (10 pts) A rope is to be hung between two poles 60 feet apart. If the rope assumes the shape of the function $y = 10 + 15(e^{\frac{x}{30}} + e^{-\frac{x}{30}})$, compute the length of the rope.

Part B

6. (20 pts)

Give a careful sketch of the parametric curves given below on the axes provided. Label the point on each curve where $t = 0$ and put arrows on the curve indicating the direction of travel along the curve as t increases.

$$\begin{cases} x &= 3t \\ y &= 1 + t \end{cases}$$

$$\begin{cases} x &= -\cos(t) \\ y &= \sin(t) \end{cases}$$

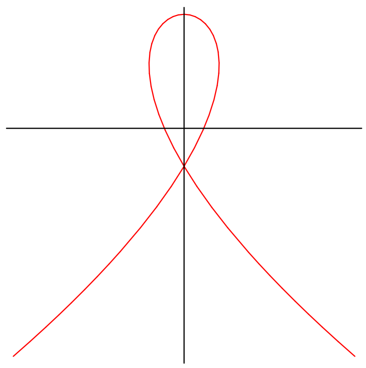
$$\begin{cases} x &= \sin(t) \\ y &= 1 + \sin(t) \end{cases}$$

$$\begin{cases} x &= 3t^2 \\ y &= 1 + t^2 \end{cases}$$

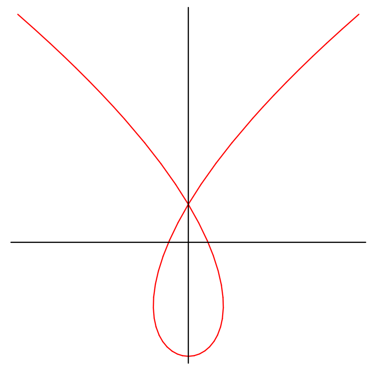
7. (15 pts) (a) Sketch the polar curve $r = \cos(2\theta)$.

(b) Find the area inside the polar curve $r = \cos(2\theta)$.

8. (10 pts) The graph of the parametric curve $\begin{cases} x = t^2 - 3 \\ y = t^3 - 4t \end{cases}$ forms a loop. Circle the correct graph of the loop from the four graphs below and write BUT DO NOT EVALUATE an integral which calculates the area inside the loop.



1



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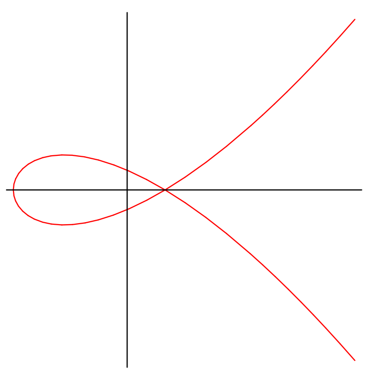

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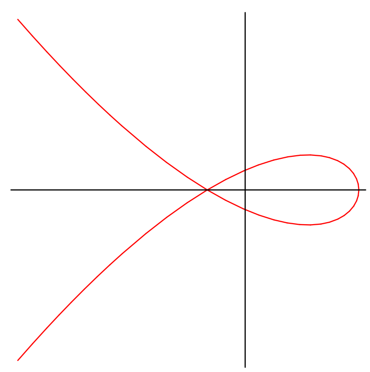

4

AREA =

9. (15 pts) For each of the series given below determine whether or not it is **convergent or divergent.** If it is convergent, calculate the **sum** of the series.

(a) $\frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{8} + \frac{\pi}{16} + \frac{\pi}{32} + \dots$

ANSWER: ____________________

(b) $\sum_{n=3}^{\infty} (-1)^n \frac{\pi^{2n}}{9^n}$

ANSWER: ____________________

(c) $\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$

Hint: Use partial fractions

ANSWER: ____________________

10. (20 pts) Determine whether the series **converges** or **diverges.** State the name of the test you are using, check the hypotheses of the test, and clearly state your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2+n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{4+n^2}{9+100n^2}$

(d) $\displaystyle\sum_{n=6}^{\infty} \frac{(3n)100^n}{n!}$

11. (10 pts) (a) Find the 3^{rd} degree Taylor polynomial, $T_3(x)$, for $f(x) = \sqrt[3]{x}$ centered at $a = 8$. You do **NOT** need to simplify your answer by multiplying out the fractions.

(b) Use the third degree Taylor polynomial to approximate the value of $\sqrt[3]{7.6}$.

12. (10 pts)

(a)Use power series to evaluate the integral below. Explain why it is necessary to use power series to evaluate this integral.

$$\int \cos(x^3)\, dx$$

(b) Evaluate the integral $\int_0^{0.1} \cos(x^3)\, dx$ using power series, that is write down the precise value in terms of a series.

(c) Write an approximate value for $\int_0^{0.1} \cos(x^3)\, dx$ so that the error in your approximation is less than 10^{-25}. Explain why your error is less than 10^{-25}.

Formulas:

$$\sum_{i=1}^{n} a = a \cdot n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\begin{aligned} \sin^2 x &= \frac{1}{2}\big(1-\cos(2x)\big) \\ \cos^2 x &= \frac{1}{2}\big(1+\cos(2x)\big) \end{aligned}$$

Expression	Substitution	
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

$$\int_{\alpha}^{\beta} \frac{1}{2} r^2 \; d\theta$$

$$\int_{a}^{b} \sqrt{1 + (f'(x))^2} \; dx$$

$$\int \sec(x) \; dx = \ln|\sec(x) + \tan(x)| + C$$