MATH 142

Final Exam

December 14, 2004

NAME (please print legibly): _____

Your University ID Number: _____

- **The first part of the final can replace your lowest midterm score, but it will also count towards your score on the final. If you skip it, you will get at most 100 points out of 200.**
- **No calculators are allowed on this exam.**
- **Please put your final answers in the spaces provided.**
- **When integrating, put down all information you are using, such as substitutions or integration by parts.**
- **You do not need to simplify expressions such as $\frac{3}{4^2} + 26(3)^4 - \frac{\pi}{2}$, but you do need to evaluate expressions such as $\sin(\pi/4)$.**

Part A		
QUESTION	VALUE	SCORE
1	40	
2	20	
3	10	
4	15	
5	15	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
6	20	
7	35	
8	15	
9	15	
10	15	
TOTAL	100	

Part A
Formulas:

$$\begin{aligned}\sin^2 x &= \frac{1}{2}\big(1-\cos(2x)\big)\\ \cos^2 x &= \frac{1}{2}\big(1+\cos(2x)\big)\end{aligned}$$

Expression	Substitution	
$\sqrt{a^2-x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2+x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2-a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

$$\int_\alpha^\beta \frac{1}{2}r^2\ d\theta$$

$$\int_a^b \sqrt{1+(f'(x))^2}\ dx$$

$$\int \sec(x)\ dx = \ln|\sec(x)+\tan(x)| + C$$

1. (40 pts) Evaluate the following integrals.

(a) $\int \cos^3(x)\sin^2(x)\,dx$

ANSWER: _____

(b) $\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$

ANSWER: _____

(c) $\displaystyle\int_0^4 \frac{\ln(x)}{\sqrt{x}}\, dx$

ANSWER: _____

(d) $\displaystyle\int \frac{x^2 + 8x - 3}{x^3 + 3x^2}\, dx$

ANSWER: _____

2. (20 pts) Consider the region R bounded by the curves

$$y = x, \qquad y = -\frac{1}{2}x + 3, \qquad y = 1.$$

(a) Sketch the curves given above, label the points of intersection, and shade the region.

(b) Set up but DO NOT EVALUATE an integral which calculates the *volume* of the solid obtained by revolving the region about the x-axis.

(c) Set up but DO NOT EVALUATE an integral which calculates the *volume* of the solid obtained by revolving the region about the line $y = -2$.

(d) Set up but DO NOT EVALUATE an integral which calculates the *volume* of the solid obtained by revolving the region about the line $x = 0$.

3. (10 pts) Set up, but DO NOT EVALUATE, an integral which calculates the work required to empty the water in the tank pictured below through the pictured spigot. Recall that the density of water is $1000kg/m^3$ and the acceleration due to gravity is $9.8m/s^2$.

ANSWER: _____

4. (15 pts)

Give a careful sketch of the parametric curves given below on the axes provided. Label the point on each curve where $t = 0$ and put arrows on the curve indicating the direction of travel along the curve as t increases.

$$\begin{cases} x &= -\cos(t) \\ y &= \sin(t) \end{cases}$$

$$\begin{cases} x &= 3\cos(t) \\ y &= 3\sin(t) \end{cases}$$

$$\begin{cases} x &= -t \\ y &= 1+t \end{cases}$$

$$\begin{cases} x &= t^2 \\ y &= 1-t^2 \end{cases}$$

5. (15 pts) (a) Sketch the polar curve $r = \sin(3\theta)$.

(b) Find the area inside one petal of the polar curve $r = \sin(3\theta)$.

Part B

6. (20 pts) For each of the series given below determine whether or not it is **convergent or divergent.** If it is convergent, calculate the **sum** of the series.

(a) $15 + 10 + \frac{20}{3} + \frac{40}{9} + \frac{80}{27} + \dots$

ANSWER: _____

(b) $\sum_{n=3}^{\infty} (-1)^n \frac{3^{2n}}{10^n}$

ANSWER: _____

(c) $\sum_{n=1}^{\infty} \frac{10^n}{9^{n+2}}$

ANSWER: _____

(d) $\sum_{n=1}^{\infty} \frac{2}{n(n+1)}$ Hint: Use partial fractions to rewrite $\frac{2}{n(n+1)}$ and then use the definition of what it means to sum infinitely many terms.

ANSWER: _____

7. (35 pts) Determine whether the series converges or diverges. State the convergence test you are using, check the hypotheses of the test, and clearly state your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n} + 4n^6}$

ANSWER: _____

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{10 + 3^n}{2 + 4^n}$

ANSWER: _____

(c) $\displaystyle\sum_{n=6}^{\infty} \frac{\ln(n)}{100n}$

ANSWER: _____

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2-3n}{9+n}$

ANSWER: _____

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 7}{\sqrt{n+3}}$

ANSWER: _____

8. (15 pts) Find the radius and interval of convergence for the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!} x^n$

ANSWER: _____

(problem 8 continued, find radius and interval of convergence.)

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n \sqrt[3]{n}}(x+6)^n$

ANSWER: _____

9. (15 pts)

(a) Find a power series representation, centered at 0, for the function below, and determine its **interval** of convergence. $f(x) = \dfrac{1}{1+x^2}$

ANSWER: _____

(b) Use the power series found in (a) to find a power series representation for

$g(x) = \arctan(x)$, and SHOW that its interval of convergence is $[-1, 1]$.

ANSWER: _____

(c) Give an approximation of the value of the integral below with error less than $(.3)^{22}$.

$$\int_0^{0.3} \arctan(x^3)\ dx$$

ANSWER: _____

10. (15 pts) (a) Find the 3^{rd} degree Taylor polynomial, $T_3(x)$, for $f(x) = \sqrt{x}$ centered at $a = 4$. You do **NOT** need to simplify your answer by multiplying out the fractions.

ANSWER: _____

(b) Use the third degree Taylor polynomial to approximate the value of $\sqrt{4.2}$. Give a bound on the error of your approximation.

ANSWER: _____