Group Exam 4

Calculus II
Professor Johnson
Fall 2008

Name: ______________________
Name of group member: ______________________
Name of group member: ______________________

Problem 1: We want to evaluate the integral $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, dx$.

a) Why is this an improper integral?

b) Show that $\sqrt{\frac{1+x}{1-x}} = \frac{1+x}{\sqrt{1-x^2}}$ if $-1 < x < 1$.

c) Use part b) to evaluate the indefinite integral $\int \sqrt{\frac{1+x}{1-x}}\, dx$.

d) Use the antiderivative in part c) to evaluate $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, dx$.

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Problem 2: Let R be the region between the curves $y = x + \frac{1}{x^2}$ and $y = x - \frac{1}{x^2}$ over the interval $[1, \infty)$. Set up an integral that calculates the area of this region. Find the area of the region R, if it is finite. If it is not finite, show why.

Consider the solid obtained by rotating the region R about the x-axis. Set up an integral that calculates the volume of this solid. Find the volume of this solid, if finite. If this volume is not finite, show why.

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Problem 3: **The Comparison Test for Improper Integrals**
Consider the picture below of the positive functions f, g, and h.

a) If $\int_a^\infty g(x)\ dx$ is convergent, then can you conclude anything about $\int_a^\infty f(x)dx$? If so, what can you conclude and why?

b) If $\int_a^\infty g(x)\ dx$ is divergent, then can you conclude anything about $\int_a^\infty h(x)dx$? If so, what can you conclude and why?

c) The comparison ideas found above can be used to determine whether an improper integral is convergent or divergent, even if we cannot calculate the integral explicitly. For example, consider the integral $\displaystyle\int_1^\infty \frac{1}{x+e^x}\ dx$.
Convince yourself that all the tools in your integration toolbox fail to help you evaluate this integral. However, the following two inequalities hold for all $x \geq 1$.
$0 \leq \dfrac{1}{x+e^x} \leq \dfrac{1}{x}$ and $0 \leq \dfrac{1}{x+e^x} \leq \dfrac{1}{e^x}$ Explain why these inequalities are true.

d) One of the two inequalities in part c), along with your conclusions in parts a) - b), can be used to determine whether the integral $\displaystyle\int_1^\infty \frac{1}{x+e^x}\ dx$ is convergent or divergent. **Circle** the useful inequality, **state** whether the integral in question is convergent or divergent, and **explain** how the inequality is used to make your conclusion regarding convergence/divergence.