

Group Exam 1

Name: _____

Math 142, 11:30AM

Name of group member: _____

Professor Johnson

Name of group member: _____

Problem 1: Recall the definition of the definite integral for a continuous function $f(x)$ on the interval $[a, b]$.

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x, \quad \text{where } \Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i \Delta x$$

[1.1] Describe in **1-2 sentences** the meaning of the quantity $f(x_i) \Delta x$ in this definition. You may assume $f(x) \geq 0$ and you may draw a picture in the margin to assist you in your description.

[1.2] Describe in **1-2 sentences** the meaning of the notation $\lim_{n \rightarrow \infty} \sum_{i=1}^n$ in this definition.

[1.3] Using the definition of the integral above, (1) identify the following limit of sums as an integral, and (2) calculate the integral using the Fundamental Theorem of Calculus.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(4 \left(2 + i \left(\frac{3}{n} \right) \right)^6 - 7 \left(2 + i \left(\frac{3}{n} \right) \right) \right) \cdot \left(\frac{3}{n} \right) =$$

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Problem 2: Fill in the blanks in the statements below which describe how the derivative affects the shape of the graph of a function.

[2.1] Suppose $f(x)$ is a differentiable function on the interval (a, b) .

If $f'(x) > 0$ on (a, b) , then the function $f(x)$ is _____ on (a, b) .

If $f'(x) < 0$ on (a, b) , then the function $f(x)$ is _____ on (a, b) .

If $f''(x) > 0$ on (a, b) , then the function $f(x)$ is _____ on (a, b) .

If $f''(x) < 0$ on (a, b) , then the function $f(x)$ is _____ on (a, b) .

[2.2] Match the function to the corresponding graph below which most closely resembles the graph of the function. Use the first and second derivatives to justify your selection, and write **1-2 sentences** explaining your choice.

$$g(x) = x^5 - 5x$$

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Problem 3: Suppose that, for a particular population of organisms, the birth rate is given by $b(t) = 410 - 0.3t$ and the death rate is given by $g(t) = 390 + 0.2t$ organisms per month.

[3.1] Calculate $\int_0^{12} [b(t) - g(t)] dt$.

[3.2] What are the units of quantity you calculated in [3.1] ? And what does this number represent?

[3.3] Determine the values of t for which $b(t) > g(t)$.

[3.4] Answer the following questions in complete **sentences** and explain why the calculation(s) above justify your solutions.

At which times is the population increasing? At which times is the population decreasing? At what time does the population reaches a maximum?

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