Group Exam 2

Name: ______________________________

Math 142, 11:30AM

Name of group member: ______________________________

Professor Johnson

Name of group member: ______________________________

Problem 1:
(a) Find the volume of the solid obtained by rotating the region bounded by $y = \dfrac{1}{\sqrt{x+1}}$, $0 \leq x \leq 1$, about the x-axis.

(b) Find the volume of the solid obtained by rotating the region bounded by $y = \dfrac{1}{1+x^2}$, $0 \leq x \leq 3$, about the y-axis.

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Problem 2:
The secant function is not one-to-one, so its domain is restricted to $[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$ in order to define the inverse secant function.

$\sec^{-1}(x) = y$	means	$\sec(y) = x$ and, $0 \le y < \frac{\pi}{2}$ or $\pi \le y < \frac{3\pi}{2}$

Follow the steps below to find the derivative of the $\sec^{-1}(x)$ function.

Let $y = \sec^{-1}(x)$, then we *want* to find a formula for $\frac{dy}{dx}$. By the definition of $\sec^{-1}(x)$ given above, y is an angle in one of the two intervals above and

$$\sec(y) = x \qquad [1]$$

(a) Use the chain rule to take the derivative of formula [1] above, then find a formula for $\frac{dy}{dx}$. (Note, your answer here will have a y in it, but we will find a formula with only x's in it by following the steps below.)

(b) Label all three sides of the right triangle to reflect the fact that $\sec(y) = x = \frac{x}{1}$.

(c) Use the triangle above to find the formula for $\frac{dy}{dx}$ in terms of x.

$$\frac{d}{dx}(\sec^{-1}(x)) = \frac{dy}{dx} = \underline{\hspace{6cm}}$$

(e) Calculate $\frac{d}{dx}\left(\sqrt{\sec^{-1}(9x)}\right)$

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Problem 3:
(a) Draw the graph of $f(x) = \sin^{-1}(x)$. [Hint: first draw a graph of $y = \sin(x)$ with the appropriate restricted domain.]

(a.1) The integral $\int_0^1 \sin^{-1}(x)\ dx$ represents an area in your picture above. Shade the area it represents.

(a.2) Now calculate the shaded area by slicing the region parallel to the x-axis, i.e. thickness Δy, and setting up the appropriate integral with respect to y.

(b) Calculate $\int_{\sqrt[3]{0.5}}^{1} \frac{t^2}{\sqrt{1-t^6}}\ dt$ (Hint: what is the derivative of $\sin^{-1}(t)$?)

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