MATH 162

Midterm 1

October 2, 2003

NAME (please print legibly): _____

Your University ID Number: _____

Circle your Instructor's Name:

Inga Johnson Steve Gonek

- **No calculators are allowed on this exam.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your final answers in the spaces provided.**
- **When integrating, put down all information you are using, such as substitutions or integration by parts.**

Formulas:

$$\sin^2 x = \frac{1}{2}\big(1 - \cos(2x)\big)$$
$$\cos^2 x = \frac{1}{2}\big(1 + \cos(2x)\big)$$

Expression	Substitution		Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$	$1 - \sin^2\theta = \cos^2\theta$

QUESTION	VALUE	SCORE
1	24	
2	24	
3	40	
4	12	
TOTAL	100	

1. (24 pts) Calculate the integrals.

1. $\int x^5 \sqrt[3]{2+x^3}\, dx$

2. $\int_0^{\frac{\pi}{6}} \sin^3(x)\, dx$

2. (24 pts) Calculate the integrals.

1. $\int (\ln x)^2 \, dx$

2. $\int \frac{1}{x^2\sqrt{x^2-1}} \, dx$

3. (40 pts) Consider the region, R which is bounded between the curves

$$\begin{aligned} y &= x+2 \\ y &= (x-4)^2 \end{aligned}$$

1. Sketch the region. Label the points of intersection.

2. Write an integral for the volume of the solid formed by rotating this region R about the x-axis. DO NOT evaluate the integral.

3. Write an integral for the volume of the solid formed by rotating this region R about the y-axis. DO NOT evaluate the integral.

4. Write an integral for the volume of the solid formed by rotating this region R about the line $x = -2$. DO NOT evaluate the integral.

4. (12 pts)

A trough is 10 feet long and 9 feet high. The vertical cross-section of the trough parallel to an end is shaped like the graph of $y = x^2$. The trough has 7 feet of mercury in it with a weight of $200 lbs/ft^3$. Set up *but do not evaluate* an integral for the amount of work in foot-pounds required to empty the trough by pumping the mercury over the top.