Math 256: Homework Assignment #3

1. Integration by Partial Fraction decomposition.

   (a) Consider the differential equation $\frac{dP}{dt} = (2P)(1 - \frac{P}{10})$. Explain the relationship between this differential equation and the integral $\int \frac{1}{2P(1-\frac{P}{10})} dP$.

   (b) Find the values of A and B such that

$$\frac{1}{2P(1-\frac{P}{10})} = \frac{A}{2P} + \frac{B}{1-\frac{P}{10}}.$$

   (c) Use the decomposition in (b) to evaluate the integral in (a).

2. Do problem 24 part (c) from section 1.3. (this problem was postponed from last week) Hint: use partial fraction decomposition!

3. Taylor polynomials. (a) Let $f(x)$ be a function with $n+1$ derivatives at $x = a$. The n^{th} *dergree Taylor polynomial for f centered at $x = a$* is a polynomial, $T_n(x)$, of degree n which satisfies the following conditions:

   $T_n(a) = f(a),\ \ T_n'(a) = f'(a),\ \ T_n''(a) = f''(a),\ \ \ldots,\ T_n^{(n)}(a) = f^{(n)}(a).$

   Suppose

$$T_n(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \ldots + c_n(x-a)^n \qquad (*)$$

   Prove that the coefficients c_n are given by the formula

$$c_i = \frac{f^{(i)}(a)}{i!}.$$

   Hint: plug a into equation (*). What information does this give you? Differentiate equation (*) and then plug in a. What information does this give you about the coefficients? Continue differentiating equation (*) and plugging in a. What is the general formula for c_i?

   (b) Find the 1st degree Taylor polynomial, $T_1(t)$, for the function $y(t)$ centered at $t = t_0$ and evaluate it at $t = t_1$. Compare this to the formula for step 1 of Euler's method, are these the same or different? explain.

4. do section 7.1 problems 10, 11*, 12*

5. (a) Use Euler's Method to approximate the initial value problem

$$\frac{dy}{dx} = -0.3x^2 + \cos(y), \qquad y(0) = 0$$

   on the interval $0 \le x \le 2$ with $\Delta x = 0.5$. Give a bound on the error of your approximation using the error estimate formula derived in class. Please show all work regarding the error estimate calculations.

   (b) Consider the initial value problem from part (a). What is the minimum value of n such that Euler's method with n steps over the interval $0 \le x \le 2$ is guaranteed to have error less than 0.04 (according to the error estimate derived in class). Please show all work.