Products of Groups!

The ***external direct product*** of the groups G_1, G_2, ..., G_n is

$$G_1 \oplus G_2 \oplus \cdots \oplus G_n = \{(g_1, g_2, \ldots, g_n) \mid g_i \in G_i\}$$

with operation

$$(g_1, g_2, \ldots, g_n) * (h_1, h_2, \ldots, h_n) = (g_1 h_1, g_2 h_2, \ldots, g_n h_n).$$

Products of Groups!

The ***external direct product*** of the groups G_1, G_2, ..., G_n is

$$G_1 \oplus G_2 \oplus \cdots \oplus G_n = \{(g_1, g_2, \ldots, g_n) \mid g_i \in G_i\}$$

with operation

$$(g_1, g_2, \ldots, g_n) * (h_1, h_2, \ldots, h_n) = (g_1 h_1, g_2 h_2, \ldots, g_n h_n).$$

Properties of external direct products:

- $G_1 \oplus G_2 \oplus \cdots \oplus G_n$ is a group.
- $|(g_1, g_2, \ldots, g_n)| = \text{lcm}(|g_1|, |g_2|, \ldots, |g_n|)$.
- $G_1 \oplus G_2 \oplus \cdots \oplus G_n$ is cyclic if and only if G_1, G_2, ..., and G_n are cyclic and $|G_1|$, $|G_2|$, ..., and $|G_n|$ are relatively prime.