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- Common notation: $*$ as *addition* $a + b$, with identity 0 and inverses $-a$, or $*$ as *multiplication* ab, with identity 1 and inverses a^{-1}.