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The **factor ring** R/A is the set $\{r + A \mid r \in R\}$ with operations
 $(s + A) + (t + A) = s + t + A$ and $(s + A)(t + A) = st + A$.

Theorem

For a subring A of R , R/A is a ring if and only if A is an ideal of R .