Integral Domains!

Let R be a commutative ring.

A ***zero-divisor*** in R is a nonzero element $a \in R$ such that $ab = 0$ for some nonzero element $b \in R$.

An ***integral domain*** is a commutative ring with unity and no zero-divisors.

Integral Domains!

Let R be a commutative ring.

A ***zero-divisor*** in R is a nonzero element $a \in R$ such that $ab = 0$ for some nonzero element $b \in R$.

An ***integral domain*** is a commutative ring with unity and no zero-divisors.

The ***characteristic*** of a ring R, denoted char R, is the smallest positive integer n such that $na = 0$ for all $a \in R$. If no such integer exists, we say char $R = 0$.

Properties of Integral Domains

Theorem (Cancellation Law)

Let R be an integral domain and $a, b, c \in R$. If $a \neq 0$ and $ab = ac$, then $b = c$.

Theorem

A finite integral domain is a field.

Theorem

Let R be a ring with unity 1. If 1 has infinite additive order, then $\text{char}\, R = 0$. If 1 has order n under addition, then $\text{char}\, R = n$.

Theorem

The characteristic of an integral domain is 0 or prime.