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The ***symmetric group*** S_n is the group of all permutations of the set $\{1, 2, \ldots, n\}$.

Properties of permutations

Theorem

Every permutation can be written as a product of disjoint cycles.

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Disjoint cycles commute.

Theorem

The order of a product of disjoint cycles is the least common multiple of the lengths of the cycles.

The alternating group

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$|A_n| = n!/2$.