★ Rings! ★

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A ***ring*** is a non-empty set R together with two binary operations "addition" $a + b$ and "multiplication" ab on R that satisfy

1. **Addition is commutative.** $a + b = b + a$ for all $a, b \in R$.
2. **Addition is associative.** $(a + b) + c = a + (b + c)$ for $a, b, c \in R$.
3. **Additive identity.** There exists an element $0 \in R$ such that $a + 0 = 0 + a = a$ for all $a \in R$.
4. **Additive inverses.** For all $a \in R$, there exists an element $b \in R$ such that $a + b = b + a = 0$.

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5. **Multiplication is associative.** $(ab)c=a(bc)$ for all $a,b,c\in R$.
6. **Distributive laws.** $a(b+c)=ab+ac$ and $(b+c)a=ba+ca$.

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- If multiplication is commutative, R is a ***commutative ring***.
- If R has a multiplicative identity, it's called a ***unity***.
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- If R is commutative, has unity, and every non-zero element is a unit, then R is a ***field***.

Properties of Rings

Let R be a ring and let $a, b, c \in R$. Let $-a$ denote the additive inverse of a.

1. $a0 = 0a = 0$.
2. $a(-b) = (-a)b = -(ab)$.
3. $(-a)(-b) = ab$.
4. $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.
5. If R has unity 1, then $(-1)a = -a$.
6. If R has unity, it's unique.
7. If a is a unit, its multiplicative inverse a^{-1} is unique.
8. **Subring Test.** Let $S \subseteq R$. S is a ring with the operations of R if for all $a, b \in S$, $a - b \in S$ and $ab \in S$.