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Theorem (Finite Subgroup Test)

Let G be a group and H be a non-empty finite subset of G.
If $ab \in H$ for all $a, b \in H$ then $H \leq G$.

Subgroups everywhere!

“Examples” of Subgroups of a Group G:

1. The subgroup ***generated by*** an element $a \in G$:
   $\langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$.

2. The intersection of two subgroups H and K of G:
   $H \cap K = \{g \mid g \in H, g \in K\}$.

3. If G is Abelian, the product of two subgroups H and K of G:
   $HK = \{hk \mid h \in H, k \in K\}$.

4. The ***center*** of G:
   $Z(G) = \{a \in G \mid ag = ga \text{ for all } g \in G\}$.

5. The ***centralizer*** of an element $a \in G$:
   $C(a) = \{g \in G \mid ag = ga\}$.