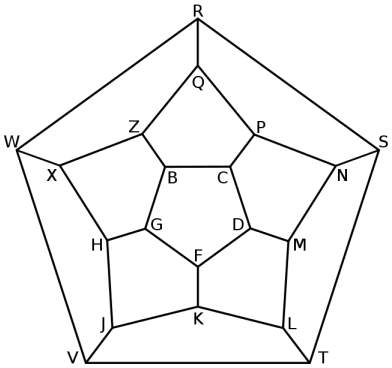
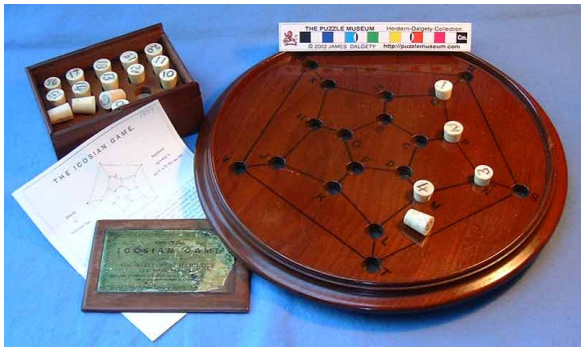
# Math 130: Contemporary Mathematics
# Spring 24
# Problem Assignment 1: Graph Theory

I encourage you to discuss these problems with me, your classmates, and math tutors. You are free to use calculators and computational tools, but not internet searches. Write your solutions on your own. Use complete sentences, and explain all your work as if to a skeptical friend. Particularly innovative, creative, or unique solutions are worth extra credit.

1. **The Icosian Game.** Here is Hamilton's Icosian Game, the reason Hamiltonian cycles are named after him. Solve these challenges that Hamilton posed for his game.




   (a) Find a Hamiltonian cycle (using each vertex once) starting with JVTSR and ending back at J.

   (b) Find a path (using each vertex once, not ending where you start) starting with BCDM, using 10 total vertices including BCDM, and getting stuck (so the path can't be extended to an 11th vertex).

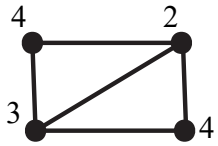
   (c) Design your own puzzle in the style of parts (a) and (b). Explain a solution to your puzzle and why you think it's a good puzzle.

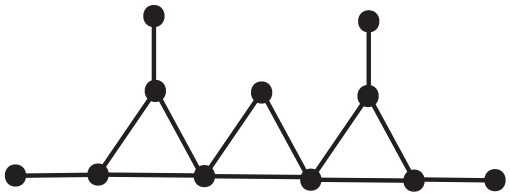
2. **Three edges everywhere.** The dodecahedron graph pictured in problem 1 is a graph with 20 vertices, all of which have 3 edges. For which numbers of vertices $n$ between 1 and 20 is it possible to have a graph with $n$ vertices, all of which have 3 edges? For each number, if it's possible, draw a graph, and if it's not possible, explain why not.

3. **Smallest graph sums.** Label the vertices of a graph with numbers 1, 2, 3,... etc., so that vertices with an edge between them have different numbers. Your score is the sum of all the numbers on the graph. For example, the labeling shown below on the left has a score of 13. A winning score is the smallest score you can get on a particular graph. The graph on the left has a winning score of 7.

   (a) What's the winning score of the graph on the right? Explain why.

   (b) For a graph with 4 vertices, what winning scores are possible? Try different graphs with 4 vertices and see how many different winning scores you can get.



4. **A graph drawing game.** Starting with six vertices and no edges, two players take turns drawing an edge between any two vertices that didn't have an edge between them already, without crossing edges. A player loses if they make a triangle on their turn (three vertices with three edges between them). Remember that edges are allowed to curve, and they still count as edges of a triangle. What are some good strategies for this game? Is there a strategy for one of the players that works all the time?

5. **Extra credit.** Ask me a question in my office about this assignment.