Math 130: Contemporary Mathematics
Spring 24
Problem Assignment 2: Number theory

I encourage you to discuss these problems with me, your classmates, and our tutor Ash. You are free to use calculators and computational tools, but not internet searches. Write your solutions on your own. Use complete sentences, and explain all your work as if to a skeptical friend. Particularly innovative, creative, or unique solutions are worth extra credit.

1. **Strings of Ones.** Using a calculator, wolframalpha, or other computing software, experiment with factoring the numbers 11, 111, 1111, 11111, ... Which of these numbers are prime? Find patterns in the factors of these numbers when they're not prime. Note: You probably won't be able to answer this question completely. Generate as much information about the patterns in the factors of these numbers as you can!

2. **Prime differences.** Can you find a set of three numbers for which the differences between any pair of them are all prime numbers? (i.e. whichever two of the numbers you pick, subtracting one from the other gives you a prime number.) Can you find a set of four numbers with this property? Five numbers? What's the largest set of numbers with this property? Whatever largest one you find, give evidence that there aren't any more.

3. **Chicken nugget combinations.**

   (a) McDowell's fast food restaurants sell chicken nuggets in boxes of 4 and boxes of 7. Explain how you can't buy exactly 17 nuggets, but you can buy any number greater than 17.

   (b) For boxes of 5 and 8, what's the largest number of chicken nuggets you can't buy? What about for boxes of 4 and 5? What about for boxes of 6 and 9?

   (c) Try several other combinations of box sizes. Try to find a pattern in the largest number you can't buy for different box sizes, and a pattern in when there is a largest number you can't buy.

4. **Fibonacci plants.** Find a plant or fruit or vegetable in the world (on campus, at the dining hall, at the grocery store, etc., not the internet) that has a Fibonacci number spiral pattern. Take pictures of it or make a video of it, and draw the spirals on your pictures or in your video to count the Fibonacci numbers in the same way Hart did in her videos.

   Warning: If you take only one picture from the side, you probably won't be able to count the spirals from the picture, since you won't be able to tell which of them are the same spiral. Think about the counting procedure and show your spirals in a way that convinces me you've counted them accurately.

5. **Extra credit.** Ask me a question in my office about this assignment.