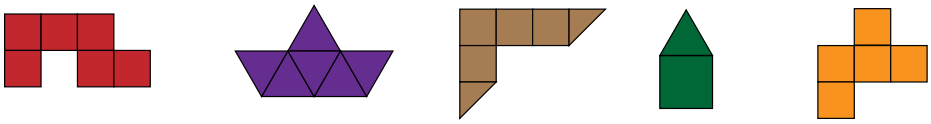
Math 130: Contemporary Mathematics
Spring 24
Problem Assignment 3: Tilings

I encourage you to discuss these problems with me, your classmates, and our tutor Ash. You are free to use calculators and computational tools, but not internet searches. Write your solutions on your own. Use complete sentences, and explain all your work as if to a skeptical friend. Particularly innovative, creative, or unique solutions are worth extra credit.

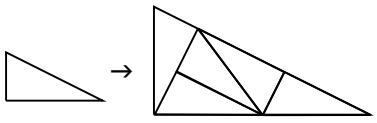
1. **Monohedral tilings.** Show that each of these shapes tiles the plane by itself, using translated, rotated, and/or reflected copies. Sketch a big enough region of the tiling so that anyone looking at your drawing can see a pattern that can continue in the same way in all directions.



2. **Pinwheel and other substitution tilings.**

   (a) The pinwheel tiling is a non-periodic tiling using single right triangle. It's not aperiodic since the triangle could tile periodically if it was arranged a different way, but it's still neat! An image of the pinwheel tiling is on the last page of this assignment.

   It's made by fitting together five copies of the triangle to get a bigger, scaled copy of the same triangle (a super-triangle). Then you can take five copies of the super-triangle to make a super-super-triangle, and keep going like this forever! Each triangle is in one unique super-triangle, and each super-triangle is in one unique super-super-triangle, etc. Its designer Charles Radin used this fact to show that the tiling is non-periodic.



   Pick a triangle in the middle of the tiling (on the last page), and outline or otherwise mark its super-triangle, super-super-triangle, and super-super-super triangle. Remember there's only one of each! Make sure you follow the rules about how the triangles fit together to get the right ones.

   (b) This website has a collection of non-periodic tilings obtained in a similar way, by making larger and larger copies of the same small pattern. Find one you like, and tell me something you like about it! https://tilings.math.uni-bielefeld.de/substitution/

3. **Making Escher tilings.** Using one of the techniques in this link, design your own Escher-like tiling. Specifically, pick one of the 4-step or 5-step instructions and accompanying animated gifs under the headings *Tessellating With Translations*, *A Two Tile Pattern*, *A cm Pattern*, or *Tessellating With Glide Reflections*.

   Use computer drawing tools or make a hand drawing. Make an image of an animal or plant or other object, not just a squiggle!

   http://euler.slu.edu/escher/index.php/Tessellations_by_Recognizable_Figures

4. **Finding wallpaper symmetries.** Use the wallpaper groups flow chart to find patterns representing at least four different types of wallpaper group occurring around your campus and/or around Salem (not just four different patterns – four different groups, i.e. different combinations of symmetries). For each pattern:

   - Give a description of where you found the pattern (e.g. on a piece of clothing, the floor or ceiling, a piece of furniture; not the internet)
   - Take a picture or make a careful sketch of the pattern
   - Describe the symmetries of the pattern, showing representative centers of rotation, axes of reflection (mirror lines), and axes of glide reflection on your photo or sketch
   - Use these symmetries to determine the pattern's wallpaper group. Write a brief explanation of how you stepped through the flow chart.

5. **Extra credit.** Ask me a question in my office about this assignment, or ask Ash a question during tutoring hours about this assignment.