Fibonacci / Virahanka / Hemachandra numbers

The numbers 1, 1, 2, 3, 5, 8, 13, 21, 34,…
You get the next number by adding the previous two numbers.

History

- Pingala, India, ~200 B.C.E.
- Virahanka, India, ~500-700
- Hemachandra, India, 1150
- Fibonacci AKA Leonardo Pisano Bigollo, Italy, 1202

Cool properties!

- Ratios of successive Fibonacci numbers approach phi.
- Continued fraction expansions!
- The exact formula

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

Things these numbers count!

1. (Pingala) The number of possible *n*-beat rhythmic poems made up of any combination of short (1 beat) and long (2 beat) syllables.

2. (Fibonacci) The number of pairs of rabbits after *n* months, starting from one pair, when each pair gives birth to a new pair every month after their first month.

3. The number of different arrangements of 1 x 2 rectangles that fill an *n* x 2 rectangle (arranged horizontally or vertically).

4. The number of different sequences of *n* Xs and Os, where you can't have two Xs in a row (using only Os is allowed).

5. The number of lists of positive odd numbers that add up to n (different orderings count as different lists, a single number counts as a list, and you can use the same number more than once).