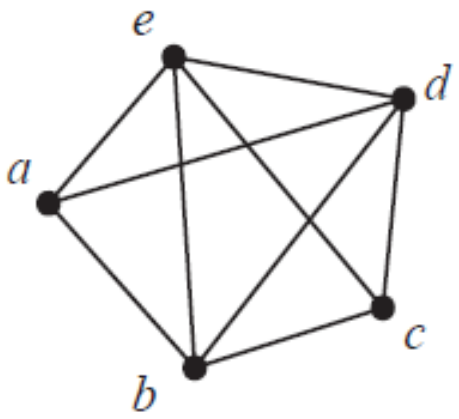
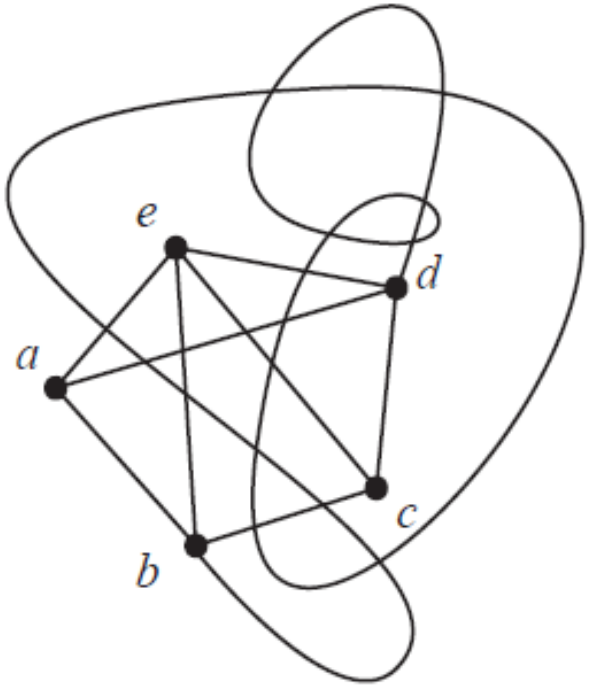
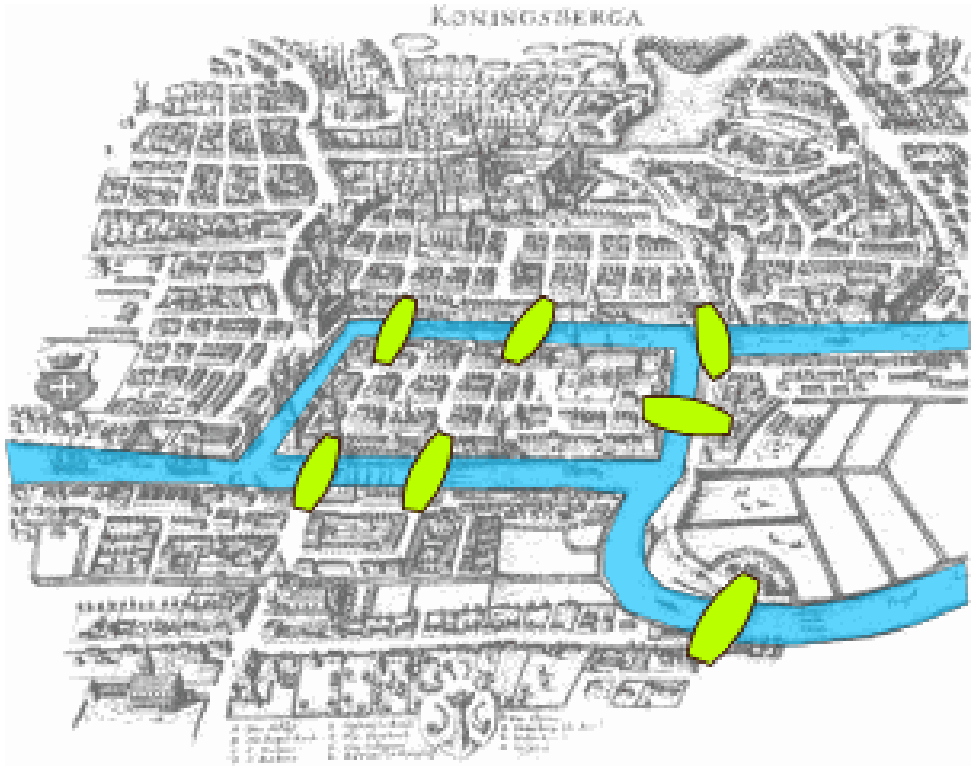
Graphs

Some points, called ***vertices***, and some line segments or curves connecting them, called ***edges***. Moving the vertices and edges around counts as the same graph.



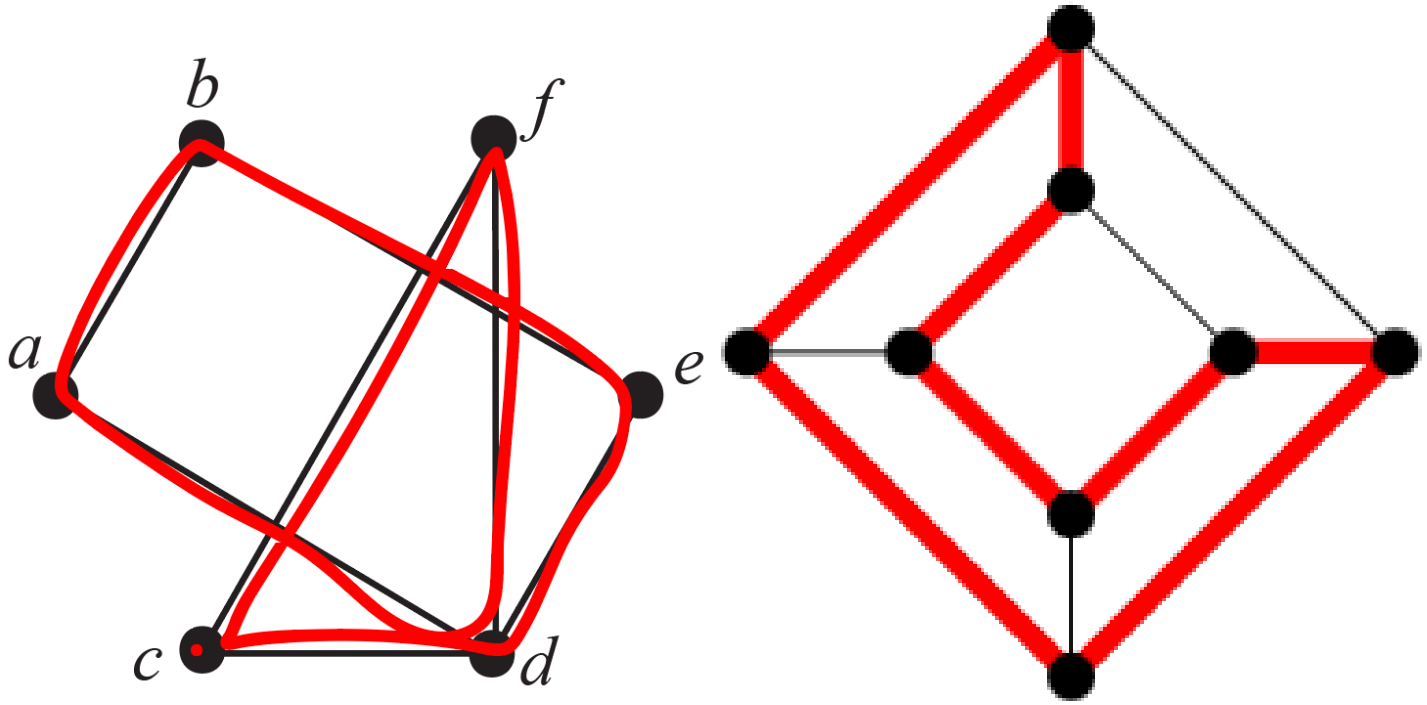




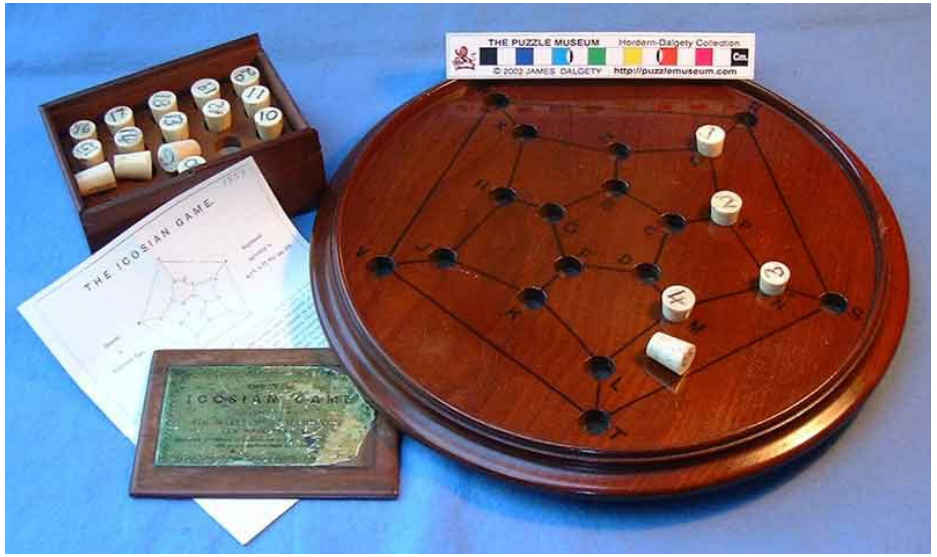
The bridges of Königsburg problem, Leonhard Euler 1736.

Circuits and cylces

An ***Eulerian circuit*** traverses each edge of a graph exactly once, coming back to the starting point.



A ***Hamiltonian cycle*** traverses each vertex of a graph exactly once, coming back to the starting point.
(William Rowan Hamilton 1857)



How many graphs are there?

Challenge: Find all the different graphs with 4 vertices.

Things to keep in mind:

- It matters whether we're thinking of labeled or unlabeled graphs. We'll count unlabeled graphs here.
- Rotating or swapping vertices doesn't change the graph.
- Some vertices might have no edges.

Types of graphs

- ***Trees*** are connected graphs with no cycles.
- ***Complete*** graphs are graphs with all possible edges.
- ***Planar*** graphs can be drawn without edges crossing.

The ***chromatic number*** of a graph is the smallest number of colors you need to color the vertices of the graph so that every two vertices with an edge between them are colored differently.

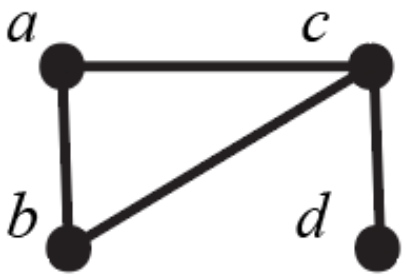
Questions:

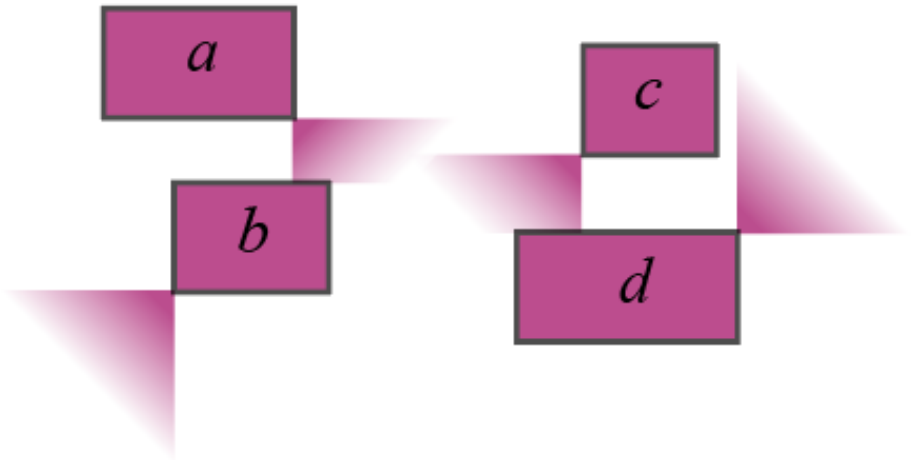
1. How many edges can each type of graph have, in terms of its number of vertices?
2. What chromatic number does each type of graph have, in terms of its number of vertices?

Corner Rectangle Visibility Graphs

Juni DeYoung, Jayden Li, Lani Southern, WU thesis spring 2023

Start with some rectangles. Each has an eye at one of its corners.
Make a graph with a vertex for each rectangle and an edge if either rectangle sees the other.
Which graphs can you make?





Polyomino Visibility Graphs

Taden Bowden, Ezekiel Druker, Chris O, Brooks Danielson, Dayton Roberts, Benjamin Weber, WU thesis spring 2024

Start with some polyominos.

Each looks left, right, up, and down from any of its sides.

Make a graph with a vertex for each polyomino and an edge if two polyominos see each other.

Which graphs can you make?