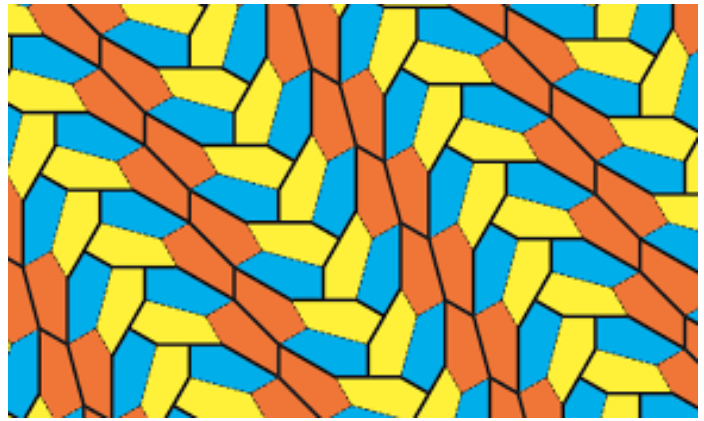
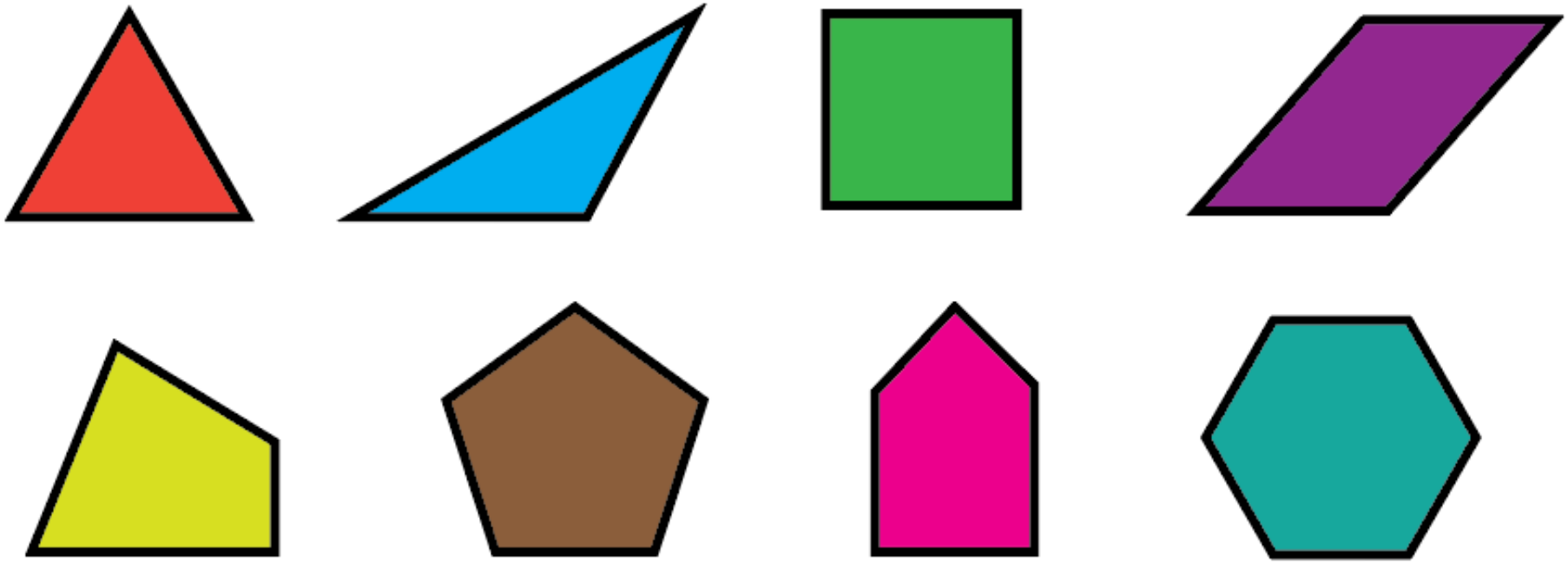
Tilings

An arrangement of some geometric shapes to cover the plane with no gaps or overlaps.

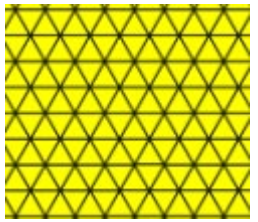


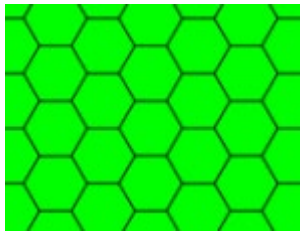
Monohedral tilings. One repeated polygon.

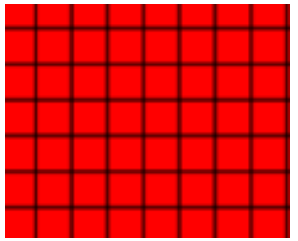
Which of these polygons work?



Tilings



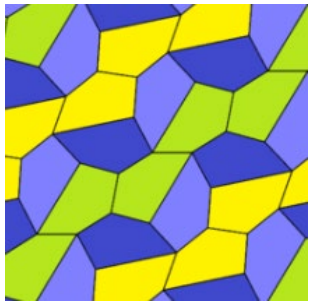


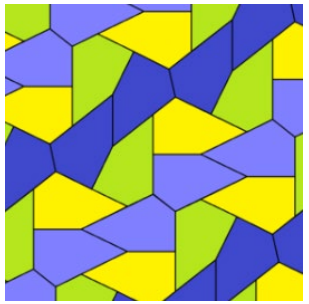


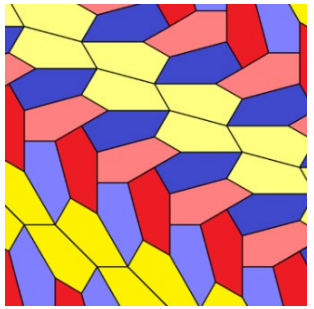
Regular tilings. One repeated regular polygon.
Triangles, squares, or hexagons only.

Monohedral tilings. One repeated polygon of any shape.
Any triangle or any quadrilateral, and 15 types of pentagon!

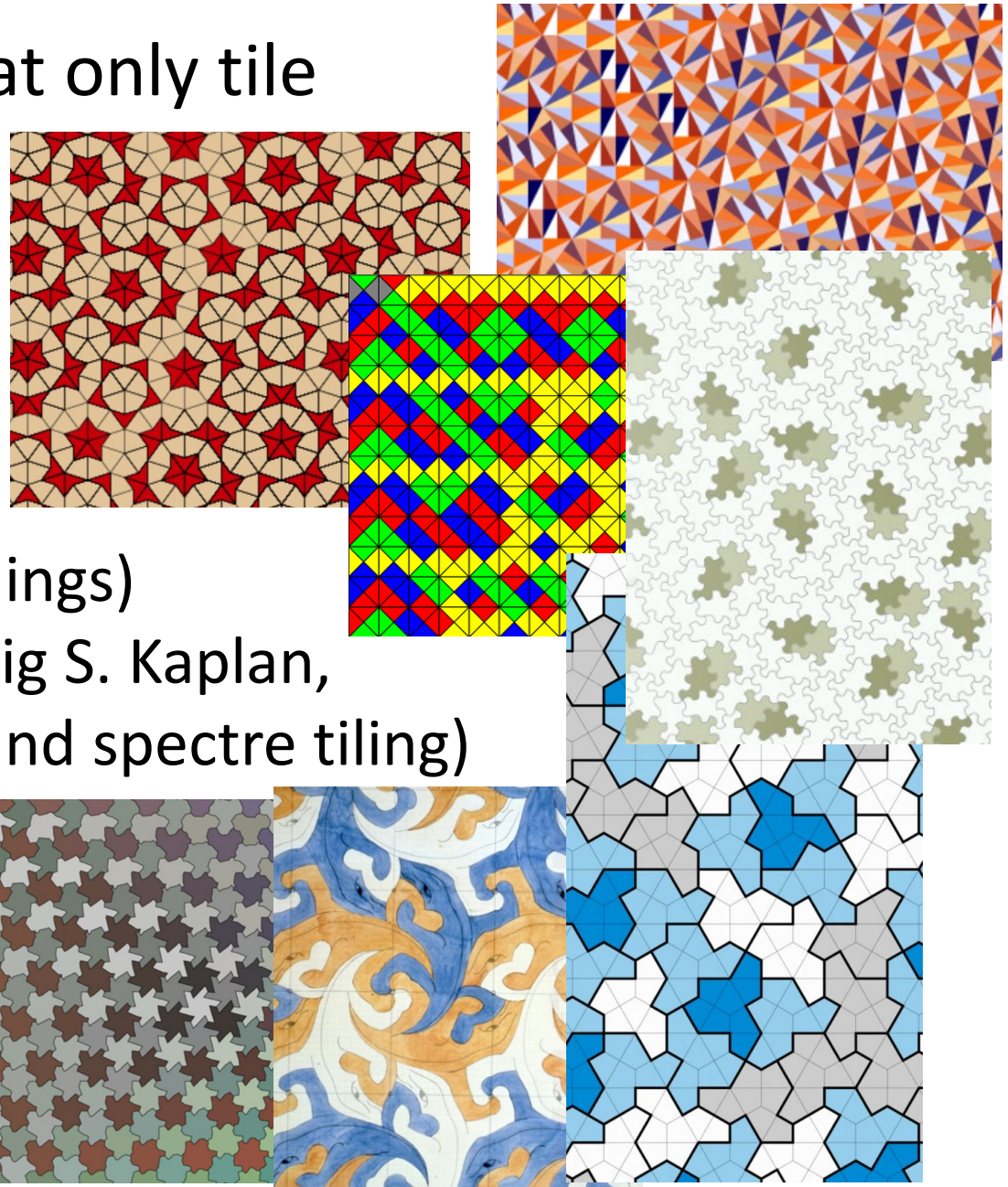
1918 Karl Reinhardt
1968 Richard Kershner
1975 Richard James
1977 Marjorie Rice
1985 Rolf Stein
2015 Casey Mann, Jennifer McLoud-Mann and David Von Derau
2017 Michael Rao







Aperiodic tilings: Tiling using shapes that only tile with no translational symmetries



1964 Hao Wang, Robert Berger, 104 shapes
1968, Donald Knuth, 94 shapes
1971, Raphael Robinson, 6 shapes
1974, Sir Roger Penrose, 2 shapes (Penrose tilings)
2023, David Smith, Joseph Samuel Myers, Craig S. Kaplan, Chaim Goodman-Strauss, 1 shape (hat tiling and spectre tiling)

Parquet deformations: Tiles that deform into other tiles.

Escher tilings: Made by artist M.C. Escher

A **wallpaper pattern** is a repeating pattern that covers the plane. It could be a tiling, or just a repeating design.

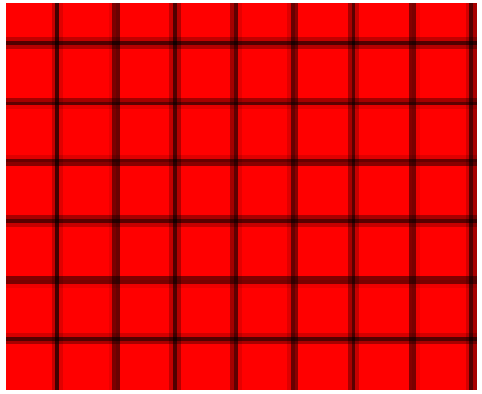
The set of symmetries of a wallpaper pattern is a **wallpaper group**. There are 17 different combinations of symmetries that work!

Wallpaper patterns have these symmetries:

- **Rotation:** around a center of rotation by some angle
- **Reflection (or flip)** about an axis of reflection
- **Translation (or shift)** by some distance in some direction
- **Glide reflection:** a reflection and a translation together, when neither the reflection nor the translation are symmetries by themselves.

Find the symmetries of these wallpaper patterns!

a)



b)

c)

d)