

Math 251: Foundations of Advanced Mathematics

Fall 2024

Syllabus

Professor: Josh Laison
he/him, Ford 215, jlaison@willamette.edu

Student Office Hours: You don't need an appointment to come see me at these times.
Monday, Wednesday, and Thursday 10:00-11:00
Friday 2:00-3:00

You are very welcome to talk with me at other times. If I'm in my office you can drop by without an appointment, or you can schedule an appointment via email to meet in person or over Zoom.

Teaching Assistant: Ash Kiel
they/them, adkiel@willamette.edu
Office hours (in the math hearth): Tuesday and Thursday, 5:30–7:00 PM

Please use the math hearth (Ford 2nd floor lounge, outside my office), my office hours, and Ash's office hours often to work together and ask questions. Using office hours is a standard part of class, not a last resort.

Class Meetings: 12:00-1:00 Monday, Wednesday, Friday, Collins 210

Textbook: Proofs and Fundamentals: A First Course in Abstract Mathematics, 1st or 2nd edition, Ethan Bloch

Course Canvas site:
<https://willamette.instructure.com/courses/5788>

Course Webpage:
<http://www.willamette.edu/~jlaison/foundations.html>

Overleaf: (Mathematical word processing software)
<https://www.overleaf.com/>

Grading:

Canvas quizzes (open notes, untimed)	10%
In-class quizzes (closed notes, 30 minutes each)	20%
Perusall reading assignments	10%
Problem assignments, first draft	10%
Problem assignments, final draft	25%
Class participation, in-class work	10%
Oral final exam	15%
Total	100%

Course Goals:

- Communicate ideas clearly in written mathematical arguments and in conversation, including proof techniques, conventional mathematical typesetting, and style.
- Improve problem-solving, logical, and analytic skills.
- Learn a core set of mathematical tools that are used in upper-level mathematics courses, related academic disciplines, and STEM industries.
- Discover and learn about some fields of theoretical mathematics.
- Gain an understanding and appreciation for the work and lives of professional mathematicians.

Canvas quizzes: I'll ask you to start thinking about new concepts and topics at the beginning of each course module, before we start talking about them in class. We'll often use the Canvas quizzes as a starting point for the new topic. They will also help you become a better reader of mathematics, a valuable skill. You are encouraged to ask questions about these before class, or work to understand them in groups with your classmates.

Problem assignments: The problems in these assignments are somewhat more challenging than the other problems in the course, and I'll ask you to think hard both about the mathematical content of the problems and the quality of your written work. Problem solutions should communicate to your reader clearly and effectively, using techniques of proof writing that we'll learn about in class.

You are encouraged to ask questions about the problem assignments, and work on them in groups with your classmates. The problems are challenging enough that you probably won't be able to do all of them by yourself without some assistance. Please write your solutions to these problems in your own words. In addition, all of your solutions to these problems should be written in the \LaTeX word-processing application.

Problem final drafts: You will edit and resubmit the problems on the problem assignments, in the week after you get them back, using feedback from Ash and me. Your grade on the problem assignments is weighted towards the final draft, to give you a chance to revise and edit your work with lower stakes.

Perusall reading assignments: These assignments will give us a chance to talk about some of the topics related to the mathematical community. I hope to share with you a sense of what it's like to be a working mathematician, and some of the thrills and challenges in the mathematical world. I'll ask you to think about and comment on these through the software Perusall. You'll also get a chance to respond to classmates' comments and start a discussion outside of class. Perusall assesses your contributions and gives you a grade. I hope this is a way for you to meaningfully engage with these materials as a community.

Participation: As a community of scholars, our work gets better when we're all invested in a common effort of learning and discovery. Please support your classmates by preparing for classes, attending classes, and actively participating in class activities and discussions.

Much of your learning and understanding of new ideas will come from thinking and talking about them in class.

Attendance at the Math Department Colloquium: According to math department policy, since you are enrolled in a 200-level mathematics course, you are required to attend at least 2 mathematics department colloquium talks. The goal of this requirement is to expose you to a wider range of mathematics, and to make you want to go to more than 2 talks! I hope you will decide by the end of the semester, as I have, that math talks are a lot of fun. If you have an inflexible scheduling conflict that prevents you from attending colloquium talks, please see me early in the semester to discuss alternatives.

Course Policies

Time Commitment: Willamette's Credit Hour Policy expects 2-3 hours of work outside of class for every hour of in-class time. Since this class meets three days a week you should anticipate spending 6-9 hours outside of class engaged in course-related activities.

Antiracism Statement (Adapted from the Office of Institutional Equity) I declare and affirm our commitment to anti-racist action in the coming academic year and beyond. I stand in solidarity with those who have been calling for justice and working for so long to end institutionalized racism and white supremacy across the country and at Willamette.

Systemic racism at Willamette University is not an issue that can be addressed superficially. It will take a deep commitment from all parts of our community to make the changes that are necessary, and that is what I offer here: a commitment to gather, build, and act on a clear anti-racist agenda together.

Land Acknowledgement: (Adapted from the Dean's Office) Willamette is built on the land of the Kalapuya, who today are represented by the Confederated Tribes of the Grand Ronde and the Confederated Tribes of the Siletz Indians. We offer gratitude for the land, for those who have stewarded it, and for the opportunity to work on it. We acknowledge that our University's history is fundamentally tied to the first colonial developments in the Willamette Valley.

Diversity and Accessible Education Statement: (Adapted from the Accessible Education Services Office) Willamette University and I value diversity and inclusion. We are committed to a climate of mutual respect and full participation. My goal is to create a learning environment that is usable, equitable, inclusive and welcoming for people of any gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, or ability.

If there are aspects of the instruction or design of this course that result in barriers to your inclusion or accurate assessment or achievement, please notify me as soon as possible. Students with disabilities are also encouraged to contact the Accessible Education Services office in Matthews 103 at 503-370-6737 or disb-info@willamette.edu to discuss a range of options to removing barriers in the course, including accommodations.

Religious Accommodations: (Adapted from the former Office of the Chaplain) Willamette University and I recognize the value of religious practice and strive to accommodate students' commitment to their religious traditions whenever possible. If you anticipate missing class for religious reasons, please contact me to discuss your needs.

Academic Integrity: (Adapted from the Dean's Office) As members of the Willamette University community, students are expected to display honesty, trust, fairness, respect, and responsibility in their academic work. Plagiarism and cheating involve intellectual dishonesty, deception, and fraud, which inhibit the honest exchange of ideas.

This course will follow Willamette University Standards of Conduct and the Willamette Ethic, described in more detail here:

http://www.willamette.edu/cla/catalog/resources/policies/plagiarism_cheating.php

Plagiarism can take different forms, but its essence is presenting the words or work of another person or source (human or artificial) as your own. When you are quoting from, paraphrasing, or using images created by another source in any of your work, you should acknowledge that source in a citation.

Topic Schedule

1. **Statements and logical operations.** Negations, converse and contrapositive, quantifiers. Sections 1.2–1.5. About 5 days of class.
2. **Proof techniques.** Direct proof, proof by contradiction, proof by contrapositive, existence proof, uniqueness proof, induction. Sections 2.2–2.5 and 6.3. About 6 days of class.
3. **Sets.** Set operations, set-builder notation, element arguments, indexed sets. Sections 3.2–3.4. About 5 days of class.
4. **Functions.** Circle diagrams, injective and surjective functions, image and inverse image, function composition, inverse functions. Sections 4.1–4.4. About 6 days of class.
5. **Relations.** Reflexive, symmetric, and transitive relations, equivalence relations and partitions. Sections 5.1 and 5.3. About 4 days of class.
6. **Introduction to number theory.** Modular arithmetic. Section 5.2. About 2 days of class.
7. **Introduction to abstract algebra.** Binary operations and groups. Sections 7.1 and 7.2. About 4 days of class.
8. **Introduction to real analysis.** Sequences and limits of sequences, convergence, increasing and decreasing sequences, alternating sequences. Section 7.8. About 3 days of class.