Theorem. *If a simple graph G with $n \geq 3$ vertices has minimum degree 2, then G has at least $2n - 3$ edges.*

Proof. We prove the statement by induction. For the base case, if G has 3 vertices and minimum degree 2, then G is K_3 and has 3 edges. Since $2n - 3 = 2(3) - 3 = 3$, G has at least $2n - 3$ edges.

For the induction step, suppose G has n vertices and minimum degree 2. By induction, G has at least $2n - 3$ edges. We add a vertex v to G to obtain a new graph G' with $n + 1$ vertices. To satisfy the hypothesis of the theorem, we must add at least two new edges incident to v. Then G' has at least $(2n - 3) + 2 = 2n - 1 = 2(n + 1) - 3$ edges, and the theorem is proved. □

Counterexample. The graph C_5 has minimum degree 2 and 5 edges, which is less than $2(5) - 3 = 7$.