Similar matrices

Two $n \times n$ matrices A and C are ***similar*** if there exists an invertible matrix P such that $C = PAP^{-1}$.

Note that A is similar to C if and only if C is similar to A.

Theorem

A and C are similar if and only if A and C are matrices for the same linear operator with different bases!

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Properties of similar matrices

1. Similar matrices have the same determinant.
2. Similar matrices have the same rank.
3. Similar matrices have the same nullity.
4. Similar matrices have the same trace.
5. Similar matrices have the same characteristic polynomial.
6. Similar matrices have the same eigenvalues.

Diagonalizable matrices

An $n \times n$ matrix is ***diagonalizable*** if it's similar to a diagonal matrix.

Theorem

Let A be an $n \times n$ matrix. The following are equivalent.

1. *A is diagonalizable.*
2. *A has n linearly independent eigenvectors.*
3. *$\mathbb{R}^n$ has a basis consisting of eigenvectors of A (an **eigenbasis**).*
4. *The sum of the dimensions of the eigenspaces of A is n.*

Theorem

If $\vec{p}_1, \ldots, \vec{p}_n$ are n linearly independent eigenvectors of A and P is the matrix with columns $\vec{p}_1, \ldots, \vec{p}_n$, then $P^{-1}AP$ is diagonal, and the diagonal entries of $P^{-1}AP$ are the corresponding eigenvalues of $\vec{p}_1, \ldots, \vec{p}_n$, in the same order!