Vectors and vector algebra

- An n-dimensional vector $\vec{v} = (v_1, v_2, \ldots, v_n)$ is both a point in n-space, and also as an arrow from the origin to that point.
- The numbers v_1, v_2, ..., v_n are called the ***components*** of $\vec{v}$.
- The ***zero vector*** $\vec{0}$ is the vector $(0, 0, \ldots, 0)$.
- A ***scalar multiple*** of a vector $\vec{v} = (v_1, \ldots, v_n)$ is $c\vec{v} = c(v_1, \ldots, v_n) = (cv_1, \ldots, cv_n)$.
- The ***sum*** of two vectors is $\vec{v} + \vec{w} = (v_1, \ldots, v_n) + (w_1, \ldots, w_n) = (v_1 + w_1, \ldots, v_n + w_n)$.
- The ***length*** or ***magnitude*** of $\vec{v} = (v_1, \ldots, v_n)$ is $||\vec{v}|| = \sqrt{v_1^2 + \cdots + v_n^2}$.
- A ***unit vector*** is a vector with length 1.
  The ***standard unit vectors*** are $\vec{i} = (1, 0)$ and $\vec{j} = (0, 1)$ in two dimensions, $\vec{i} = (1, 0, 0)$, $\vec{j} = (0, 1, 0)$, and $\vec{k} = (0, 0, 1)$ in three dimensions, and $\vec{e}_1 = (1, 0, \ldots, 0)$, $\vec{e}_2 = (0, 1, 0, \ldots, 0)$, ..., $\vec{e}_n = (0, 0, \ldots, 0, 1)$ in n dimensions.

- The ***dot product*** of $\vec{v} = (v_1, v_2, \ldots, v_n)$ and $\vec{w} = (w_1, w_2, \ldots, w_n)$ is $\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$.
- If a set of vectors $\vec{v}_1$, ..., $\vec{v}_n$ are all orthogonal to each other then they're called an orthogonal set of vectors. If they're also all unit vectors, then they're called an ***orthonormal set***.
- The vector $\vec{w}$ is a ***linear combination*** of the vectors $\vec{v}_1$ through $\vec{v}_k$ if there exist scalars c_1, ..., c_k such that $\vec{w} = c_1 \vec{v}_1 + \ldots + c_k \vec{v}_k$.

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Questions

1. Is $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ a linear combination of $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$?
2. What is the set of all linear combinations of the vector $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$?
3. What about the vectors $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$?
4. What about the vectors $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$?