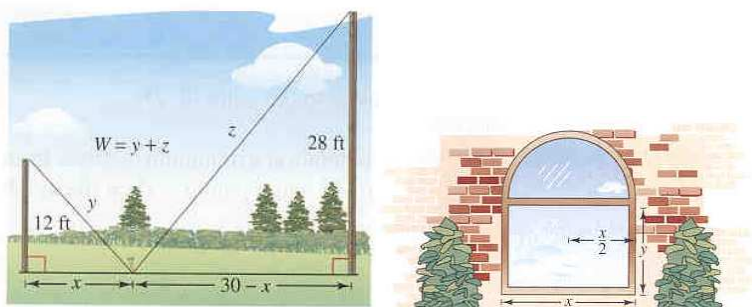


Math 141 – Calculus 1
Fall 2007
Optimization Handout

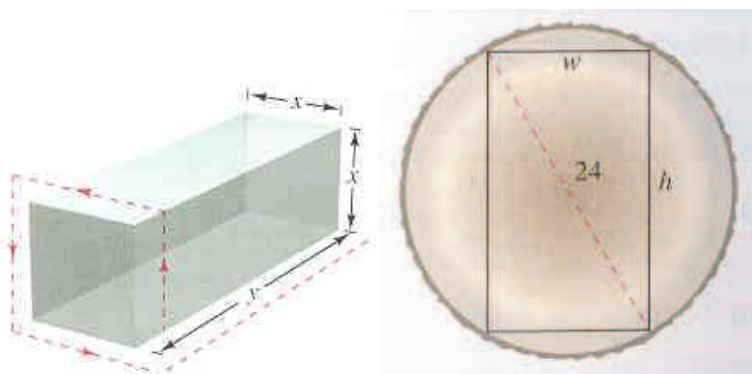
1. Josh the Farmer acquires some goats, and the goats and the sheep do not get along. He wants to fence in a rectangular area of his field, but this time he wants to divide it in half with another piece of fence to keep the sheep and goats separate. Again, he has 100 yards of fencing. What are the dimensions of the pen with the maximum area that he can build?
2. Josh sells his goats, because they're too much upkeep. But he realizes that he can make his sheep pen bigger if he uses the side of his barn for one of the four sides of the pen. With the same 100 yards of fencing, what are the dimensions of the pen with the maximum area now?
3. Four feet of wire is to be cut into two pieces. One piece will be used to form a square, and the other to form a circle. Where should the wire be cut so that the total area of the two figures is as large as possible?
4. A piece of wire 100 cm long is cut into several pieces to form the wire frame of a rectangular box with a square base.
 - (a) What are the dimensions of the box with the largest volume?
 - (b) What are the dimensions of the box with the largest surface area?
5. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be $1\frac{1}{2}$ inches, and the margins on the left and right are to be 1 inch, as shown below. What should the dimensions of the page be so that the least amount of paper is used?



6. Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post, as shown below. Where should the stake be placed to use the least wire?



7. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window, as shown below. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.
8. A rectangular package to be sent by the US postal service can have a maximum combined length and girth of 108 inches, where the girth is the perimeter of a cross section of the package. If the cross section of the package is square, as shown below, find the dimensions of the package of maximum volume that can be sent.



9. A wooden beam has a rectangular cross section of height h and width w , as shown above. The strength S of the beam is directly proportional to the width and the square of the height (i.e., $S = kh^2w$ for some constant k). What are the dimensions of the strongest beam that can be cut from a round log of diameter 24 inches?