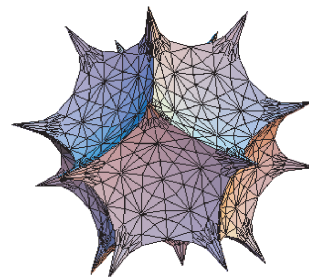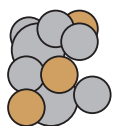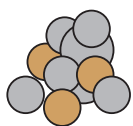# Willamette Math Problem of the Week

March 31 2008
Pocket Change

Aston and Belinda each have a small pile of (American) coins in their pocket. They have the same number of coins, and the same monetary value, but not the same combinations of coins. What's the smallest monetary value they could have?

Submit all solutions before the appearance of the next problem to Josh Laison in person, by e-mail (jlaison@willamette.edu), or by ouija board. The first correct solution gets a prize; all correct solutions get fame and glory. Preference for the prize goes to problem-solvers who haven't won one yet.

Solution to *Water into Wine*: Congratulations to Jared Nishikawa, who solved the problem and won a dashboard monk.

Suppose that each cup holds exactly 1 unit of liquid. Let the amount of water in the lower cup at a given time be $w(t)$, and the amount of wine that has poured into the lower cup so far be $v(t)$. Suppose the process starts at time $t = 0$, and finishes at time $t = 1$. We have $w(0) = 1$, $v(0) = 0$, and $v(1) = 1$. We want to find $w(1)$.

The speed at which wine enters the lower cup from above is dv/dt, and the speed at which water leaves the cup is dw/dt. The solution of water and wine leaves the lower cup at the same rate as wine enters. Since at any given time, the fraction of water in the lower cup is $w/1$, we have

$$\frac{dw}{dt} = -\left(\frac{w}{1}\right)\left(\frac{dv}{dt}\right)$$

$$\frac{dw}{w} = -dv$$

$$\int_1^{w(1)} \frac{dw}{w} = -\int_0^1 dv$$

$$\ln(w(1)) = -1$$

$$w(1) = \frac{1}{e}$$

The fraction of water in the lower cup at the end of the process is $1/e$.
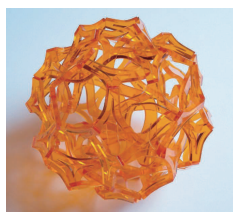
Past problems of the week, solutions, and solvers can be found at http://www.willamette.edu/~jlaison/problem.html