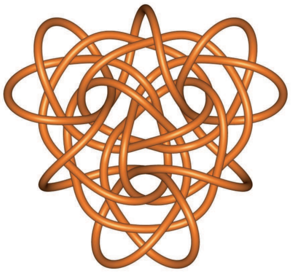
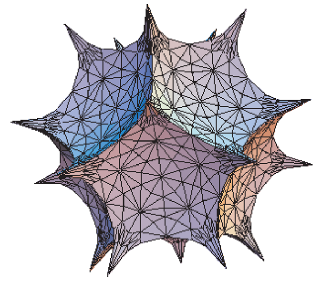
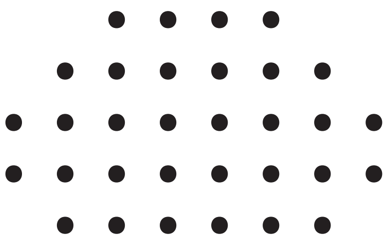


# Willamette Math Problem of the Week



## November 5 2007
## An Ice Problem



Tony Zamboni is the owner of "Zambonis For You" and their only driver. His job is to resurface the ice at the local skating rink. Wanting to impress his clients, he would like to do this as quickly as possible. He knows that if he passes through 32 key points on the ice rink, the zamboni will be wide enough to resurface every spot on the rink. These 32 points are shown in the figure (the extra width on the North side of the rink is the winners' circle for skating competitions). The zamboni enters and leaves from any two points on the South side of the rink. The problem is that Tony's vehicle doesn't turn very sharply, and can only turn 45° at each point. So he's going to have to go over some points twice. Find a zamboni path which uses as few total points as possible. (Problem due to Dennis E. Shasha)

Submit all solutions before the appearance of the next problem to Josh Laison in person, by e-mail (jlaison@willamette.edu), or by ham radio. The prize this week goes to the solution which uses the fewest points. Preference for the prize goes to problem-solvers who haven't won one yet.

**Solution to *Tower of Irrationality*:**
Congratulations to **Jason Ames**, who solved the problem and won a set of Pixie pic-up Stix, and to **Jared Nishikawa** and **Kyle Evans**, who also submitted correct solutions.

Let $x = \sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}$. Note that $x = \sqrt{2}^x$. There are two solutions to this equation (since, for example, $f(x) = x$ and $g(x) = \sqrt{2}^x$ intersect exactly twice). These two solutions are 2 and 4. But does $x = 2$ or $x = 4$?

Let $t(1) = \sqrt{2}$, $t(2) = \sqrt{2}^{\sqrt{2}}$, $t(3) = \sqrt{2}^{\sqrt{2}^{\sqrt{2}}}$, etc. So $x$ is the limit of the sequence $\{t(n)\}$. We prove that this sequence converges. Since $t(n)$ is greater than 1, and $t(n+1) = \sqrt{2}^{t(n)}$, the sequence is increasing. By induction, since $t(n) < 2$, $t(n+1) < \sqrt{2}^2 = 2$. So the sequence is bounded above by 2. Since $\{t(n)\}$ is increasing and bounded above, it converges. Since t(n) is bounded above by 2, $x = 2$ and not 4.

Past problems of the week, solutions, and solvers can be found at http://www.willamette.edu/∼jlaison/problem.html