Assignment # 23

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1.

Theorem. *If n is even then $7n$ is even.*

Proof. If n is even, it can be written in the form $n = 2k$ for some integer k. Then

$$7n = 7(2k) = 2(7k).$$

Since $7k$ is an integer, $7n$ can be written as twice an integer, so $7n$ is even. □

2. (a) $\sum_{1}^{5} n^2 = 55.$

   (b) Suppose $S = \{x \mid x \text{ is a hummingbird}\}$, $T = \{1, 2, \ldots, 7\} = \{x \mid 1 \leq x \leq 7, x \in \mathbb{Z}\}$, and $U = \{q \mid q \text{ is a bird}\}$. Then $S \subseteq U$, $U \not\subseteq S$, and $S \cap T = \varnothing$.

   (c)

   Theorem. $\int_1^2 x^3\,dx = \frac{15}{4}$

   Proof.

$$\int_1^2 x^3\,dx = \frac{x^4}{4}\Big]_1^2 \tag{1}$$

$$= \frac{16}{4} - \frac{1}{4} \tag{2}$$

$$= \frac{15}{4} \tag{3}$$

□

7. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^3$ and $g : \mathbb{R} \to \mathbb{R}$ is defined by $g(x) = \sqrt[3]{x}$ then $f \circ g(x) = x$ and $g \circ f(x) = x$, so $g = f^{-1}$.