Math 163 Discrete Math Spring 2014 Prof. Janeba

Project 2 (optional) - The Grzzmorgian Word Game

Goals are the same as in project 1, plus some experience working with directed graphs.

Another memo has arrived by government courier:

Department of State, Sub-Undersecretary of Grzzmorgian Relations
Special Office of Diplomatic Protocol and Mathematics

August 1, 2022

We thank you again for your work with the Grzzmorgian protocol problem; as has been well-reported in the media, the welcoming state dinner was a great success. The Grzzmorgians were particularly impressed when one of our diplomats let slip that we had four alternate seating arrangements we had considered, in addition to the seating arrangement we actually used.

As the world knows, relations between the Grzzmorgian envoys and the earth have been a series of discoveries. Who would have thought they would have shown such extraordinary interest in housecats? The offer of a few breeding pairs of cats was enough to "pay" for their photovoltaic technology that has quadrupled the output of our solar cells while cutting their cost in half. Similarly, the Grzzmorgians were baffled by our inability to easily synthesize edible proteins from coal and air. While many exchanges have been easy and mutually profitable, we find the Grzzmorgians are getting craftier in their trading as time passes. As always, it's hard to sort out which part of their hesitation is due to a desire to get a better deal, and which part is due to yet another incomprehensible Grzzmorgian social protocol. It's also a slow process to find out what they actually have to offer - they seem to be intentionally presenting trade items one at a time rather than all at once.

The most recent possible trade item - and we stress the word possible, as we aren't yet sure what the item is, due to the recent Grzzmorgian caginess - is an artificial gravity generator. The physicists at MIT are dubious, but Prof. Cooper from Cal Tech insists they must have one. He reasons that first, we have never seen the interior of the Grzzmorgian spacecraft, so we don't know if they had to live in a microgravity environment or not while they "coasted" in from the orbit of Uranus. Also, Dr. Cooper points out that a gravity generator would have to manipulate the fabric of space-time itself, and thus could be the basis for a "warp drive" (or "hyper drive" or "jump drive," depending on which sci-fi space opera you favor). This does explain what the exobiologists have been telling us, that the Grzzmorgians just don't seem old enough to have traveled from their home system at sub-light speeds. (We also don't know how long their trip took, but we all recall the sessile's statement about "coming of age on the green hills of Woofump." If that's to be taken literally, then their trip was not a multi-generational one. But we digress.)

Anyway, now that their crew's interactions during "shore leave" are being less closely monitored by the Grzzmorgians, we've noticed a high-ranking engineer who is particularly drawn to our casinos. We are hoping to "help" him/her/it slip up

and give us a few hints about Grzzmorgian drive technology. That's where you come in.

While this particular Grzzmorgian seems to like terrestrial casino games just fine, he (she/it) keeps trying to get someone to play something called "Thiggle." We are hoping you can help us analyze the game, and tutor some of our agents, who will pose as friendly bystanders. Whether by ingratiating themselves with the engineer, or by beating him (her/it) at his (oh futz the pronouns!) own game, we hope we can either cajole or win some information out of him.

Here are the rules of the game, as far as we can follow them (he's explained them to a host of bar patrons):

Thiggle is a Grzzmorgian "word" game, a tiny bit like Scrabble. It involves transforming "words" made out of just two letters according to four rules. The two letters (Grzzmorgian letters, of course) are "⊢" and "⊐", which we have taken to calling "T" and "U". The rules are as follows:

In a move, you may:

1. "Double" any word, provided the word is originally 6 letters or less, e.g. you can turn "⊐⊢" into "⊐⊢⊐⊢" and turn "⊢⊐⊢⊢" into "⊢⊐⊢⊢⊐⊢⊢", but you cannot use this rule on "⊢⊐⊢⊢⊐⊢⊢ ."
2. Erase any one occurrence of "⊐⊐" in the word, e.g. turn "⊐⊐⊐⊢⊢⊐" into "⊐⊢⊢⊐" and turn "⊢⊐⊐⊢⊢" into "⊢⊢⊢"
3. Replace any one occurrence of "⊢⊢⊢" with "⊐", e.g. turn "⊢⊢⊢⊢⊐" into "⊐⊢⊐" or into "⊢⊐⊢⊐" or into "⊢⊢⊐⊐".
4. Add a "⊐" at the end of any word ending in "⊢". Thus we can turn "⊢⊐⊢⊢" into "⊢⊐⊢⊢⊐", but rule 4 won't do anything with "⊢⊐⊢⊐".

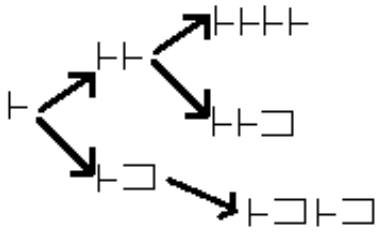
The Grzzmorgian engineer keeps trying to entice people to try this: Start with the "word" "⊢" and get to "⊐⊐⊢⊐" in twelve moves or less.

Here are the questions we would like you to help us with:

Can it be done at all (getting from "⊢" to "⊐⊐⊢⊐" in twelve moves or less)? If so, are there multiple winning "paths"? If we were allowed more "moves," can we find a sequence from "⊢" to "⊐⊐⊢⊐" that repeats part of itself, i.e. where, say, the result of the 8th move is the same as the result of the 4th move? (We want that one in case stalling appears likely to help wheedle information out of the fellow.)

What is the longest word you can get, especially if allowed extra moves? Whatever that length is, how many "words" are there of that length or less, whether or not you can make them with this game?

We have done some preliminary research into the problem, using a directed graph showing "words" we can reach and the path it takes to get there. Each vertex is a word, and there is a directed edge from one word to the next if you can make the second word out of the first using one of the rules above. For example, here is part of the graph starting at "⊢":



We think it would be particularly helpful to sketch part of the graph "working backwards" from "⊐⊐⊢⊐", i.e. showing all the words that lead to it in one step, and perhaps all the words that lead to them in turn.

We would appreciate it if you could use these directed graphs for at least part of your report.

One last thing. We think there's something special about "⊐⊢⊐"; when the Grzzmorgian engineer has been a little tipsy, he seems to mutter to himself that "all paths lead through "⊐⊢⊐".

As with your last project, we require that you explain your reasoning in sufficient detail that there is no room for doubt as to its correctness. We assume this explanation will make up the bulk of your report.

Please prepare a professional report summarizing your work and your findings, so that our agents can tell if this is a game they can play. Since your report will circulate at the highest levels of government, a rough report just won't do. Plan to spend lots of time solving the problem, and even more time carefully writing your report.

(Guidelines on the next page)

Submission guidelines:
This project is optional. If you turn it in, this project will count toward your grade the same as project #1, worth 75 class points. This is NOT extra credit, in that submitting a poorly written project report can lower your grade. If you choose not to turn in this project, your grade will be computed as if it had never been assigned. You may work in a team of one if you prefer, but groups are recommended. If you are looking for partners, contact me immediately and I will try to help people make contact with each other. Groups may not be larger than 3 people.

Report due: Friday April 25 at 4:00 p.m. at Prof. Janeba's office.
Late papers will not be accepted.

Your report will be carefully printed on plain white paper fastened with a staple. You must also submit an electronic version of your report to Prof. Janeba via email. Use Microsoft Word or other suitable word processor. Equations, as needed, will be typed. If equations are too complicated for "plain" Word, use for example the Microsoft Equation Editor built into Word. Hand-drawn graphs are acceptable if they are neat. You may wish to draw them on separate pages, numbering them as "figure 1, figure 2" etc. to facilitate references to them in your report narrative. On the front page, team member's names will be clearly visible.

The report will be self-contained, i.e. the reader need not have prior knowledge or understanding of the problem to understand your report.
In the report, your reasoning, calculations and conclusions will be fully explained in detail, as befits a professional report. Your audience for both reports includes both fellow students and the professionals at the Dept. of State. Some are quite familiar with your methods, others not very much so; none are experts in the particular problem you have solved. Explain accordingly. Your explanation and presentation are as important as the mathematics and logic, but of course your mathematical and logical analysis must be sound, complete, and correct. It should go without saying that your grammar, punctuation, and spelling will be flawless.

Group evaluation: Your group turns in one report and everyone in the group gets the same grade. To provide some accountability, however, every group member is required to submit to their instructor an informal group evaluation no later than April 29. Handwritten, typed, and email are all OK. The evaluation should include (just!) a few sentences about how well the group worked together. The evaluation must include your estimate of the share of the project work that you feel each group member did, expressed as a percentage (e.g. 33%/33%/33% or 40%/30%/30%, etc.). These percentages must add to 100%. These evaluations will be held in confidence, and used at the end of the semester, if necessary, to adjust final grades.

A word to the wise: This project, if submitted, counts for about 9% of your final grade. Please make sure that your report's qualities of completeness, clarity, and correctness reflect your best abilities. Please carefully read all the instructions and resources provided.