Math 263 Discrete Mathematics Spring 1999 Prof. Janeba Final Exam

Name_____ Score_____

Instructions: This exam is closed-book and closed-notes. Anything may be stored electronically on your calculator. Consultation is allowed ONLY with the instructor. Explain and/or show work for credit.

1. Consider arrangements of the letters in the word OSKALOOSA:

   (a) How many are there?

   (b) How many are there if the A's **must** be adjacent?

   (c) How many are there if the A's **cannot** be adjacent?

   (d) (Harder): How many arrangements of OSKALOOSA are there if no two A's can be adjacent, nor can two S's be adjacent?

2. Give the following coefficients:

   (a) The coefficient on x^4y^3 in the expansion of $(3x + 5y)^7$

   (b) The coefficient on x^2y^3z in the expansion of $(2x + 5y - z)^6$

DO ONLY ONE of #3 and #4

3. Fred must buy 12 cans of soda at a store offering separate cans in eight differenct flavors.

   (a) How many ways can Fred choose? Explain.

   (b) What if Fred is required to get at least 4 cans of Frilly Cola, which is one of the eight offered flavors?

4. A machine has an odometer-like display, for which each digit can be 0,1,2,3,4, or 5. The display has 4 digits.

   (a) How many distinct outputs can the display have?

   (b) How many of those in (a) have only digits in strictly ascending order (like 0123, 0145, or 1345, but not 1124 or 0023 or 3245)? Explain.

5. A "Wheel of Fortune" is divided into 12 symmetric and identical sectors (if confused, ask me for a drawing). Each sector is to be painted with one color, and 4 colors are available for use.

   (a) How many distinct ways can the wheel be painted?

   (b) What if the sectors are marked with numbers 1-12, and having sector 1 painted red is counted as distinct from having sector 1 painted green?

6. Prove by induction that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

7. Prove that $8^n - 1$ is (evenly) divisible by 7 for all positive integers n. Hint: You may find it helpful as part of your proof to write $8^{n+1} - 1 = 8^n \cdot 8 - 8 + 8 - 1$ and do a tiny bit of factoring.

8. Give a recursive definition of $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$. You may assume the intersection of *two* sets is already defined.

9. Convert 3274_{10} to base 5, showing work.

10. Subtract using the standard subtraction routine in base 5 (show some work):
..3273_5
-1344_5

11. Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Show at least some work for the following, and give an explicit numeric answer.

    (a) Find the number of functions from A to B.

    (b) Find the number of 1-1 functions from A to B.

    (c) Find the number of functions mapping A **onto** B.

12. Suppose that f is a function from $A = \{a, b, c, ..., z\}$ to $B = \{1, 2, 3, ..., 99\}$. Show that f maps (at least) two elements of A to numbers having the same remainder upon division by 25.

13. At a birthday party for a small child, there are a total of 5 children present. The parents hosting the party have purchased 9 distinct trinkets to give as party favors to the children (who are likewise distinct, even though they seem like a shouting blur to the beleaguered parents). Of course every child must be given at least one trinket to avoid a cataclysm, and the parents certainly don't want to have any left over. In how many ways can the trinkets be distributed?

14. Give a formal proof of **one** of the following (Be sure to clearly indicate which one!): If $f : A \to B, g : B \to C$, and...

    (a) ...f and g are both 1-1, then $g \circ f$ is 1-1.

    (b) ...f and g are both "onto", then $g \circ f$ is "onto".

    (c) ...$g \circ f$ is 1-1, then f is 1-1.

    (d) ...$g \circ f$ is "onto", then g is "onto".

Do ONE only of #15 & #16

15. Prove that if $(A, \mathcal{R})$ is a poset that is totally ordered and m is a maximal element, then m is a greatest element.

16. Prove that if $(A, \mathcal{R})$ is a poset and g is a greatest element, then $(A, \mathcal{R})$ can have no maximal elements distinct from g.

17. Fred counts the degrees of all the vertices on a polyhedron and gets 3,3,3,4,4,4, and 6, respectively. Francine says Fred can't be right. Is *she* right? Explain. Hint: how many edges would there be?

18. Francine correctly counts the degrees of all the vertices on a different polyhedron and gets 3,3,4,4,4,4, and 4, respectively. How many faces does the polyhedron have?

19. Consider the graphs G_1 and G_2.

    (a) Show that these are isomorphic.

    (b) How many isomorphisms (correspondences) between the graphs are there?

20. Solve the recurrence relation $a_{n+1} - 3a_n - 10a_{n-1} = 0$ with $a_0 = 1$ and $a_1 = 19$.