Math 256 Differential Equations Fall 2007 Prof. Janeba Exam 1A

Name_____ Score_____

Instructions: This exam is closed-book and closed-notes. Anything may be stored electronically on your calculator, except elephants, but palmtop devices are not allowed. Consultation is allowed ONLY with the instructor. Explain and/or show work for credit.

1. Solve four of the following five differential equations:

   (a) $(6x \sin y + y)\,dx + \left(x + 3x^2 \cos y\right) dy = 0$

   (b) $dy = \left(x^2 y^2 + y^2\right) dx$

(c) $xy' = 3x + 2y$

(d) $y'' = 3\left(y'\right)^2$

(e) $y''' - 4y'' + y' + 6y = 0$

2. Here are a collection of initial value problems corresponding to the differential equations in problem 1. Take a moment to note the relationship with the problems in #1, then choose two and give their solutions, using your previous work.

   (a) $(6x \sin y + y)\, dx + \left(x + 3x^2 \cos y\right) dy = 0,\ y(1) = \frac{\pi}{2}$

   (b) $dy = \left(x^2 y^2 + y^2\right) dx,\ y(0) = 1$

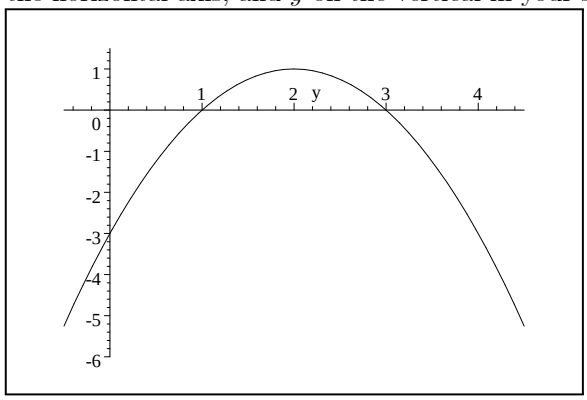
   (c) $xy' = 3x + 2y,\ y(1) = 0$

   (d) $y'' = 3\left(y'\right)^2,\ y(0) = 3, y'(0) = 1$

   (e) $y''' - 4y'' + y' + 6y = 0,\ y(0) = -2, y'(0) = 5, y''(0) = 1$

3. Solve the differential equation with initial condition: $y'' - 4y' + 4y = 72e^{-x} + 4x - 6$, $y(0) = \frac{15}{2}$, $y'(0) = -4$

4. The graph of y' as a function of y is as given (y' on the vertical axis, y on the horizontal); sketch solutions corresponding to the initial conditions $y(0) = \frac{3}{2}$, $y(0) = \frac{1}{2}$. (two solution curves) (Put t on the horizontal axis, and y on the vertical in your sketch).



5. A major component of cigarette smoke is fine particulate matter (really microscopic ash) floating in the air. One may think of smoke as a mixture of air and this particulate matter; while there are other components (e.g. carbon monoxide), we will focus on the particulate matter in this problem. Also, while smoking is now forbidden on most airline flights, that was not formerly true. In this problem, we will model the density of smoke in an airplane cabin. Suppose that the volume of the cabin is 4000 cubic feet, and that outside air is pumped in at a rate of 1200 cubic feet per minute, mixes well with interior air, and flows out at the same rate (these number approximated from Boeing publications for one 737 model). The outgoing air flushes out particulate matter from the cabin, since the outgoing air has the same particulate concentration as the cabin air at that moment in time. Further, suppose that one smoker puts particulate matter into the air at a rate of 80 milligrams per minute (that one's a wild guess, but let's go with it). Finally, suppose that at our initial time, the air in the cabin is free of particulate matter, and at that time, one smoker begins smoking.

   (a) Give a differential equation with initial condition that models the amount of particulate matter in the cabin air as a function of time (define your variables explicitly).

   (b) Modify your differential equation to model n smokers (n times the rate of particulate matter production).

   (c) For the n smoker model, the amount of particulate matter in the air approaches what equilibrium solution as $t \to \infty$? (Let's hope the flight doesn't last that long. Note you should NOT have to solve the DE to answer this problem) Give a sentence or two interpreting the meaning of this result for someone assessing the secondhand smoke exposure of an airline attendant who worked on such flights.

6. For the initial value problem $y'' - 4y' + 5y = 0, y(0) = 1, y'(0) = 3$, do one of the following two problems:

   (a) Solve the d.e. using series methods expanding about $x_0 = 0$, giving the terms of the series solution through the x^3 term, OR

   (b) Solve the d.e. using Laplace transform methods, using the provided table.

7. Using Euler's method to solve $y' = \cos(y), y(0) = 1$ we get the following estimates of $y(1)$:

Stepsize	estimate
0.1	1.366 071 742
0.05	1.360 806 967

(a) Use your calculator to find an Euler estimate with stepsize 0.025

(b) Use your estimate and the table above to estimate the error in your answer in (a). If you can't get an estimate for (a), estimate instead the error in the 0.05 estimate in the table; be clear which estimate you are giving.