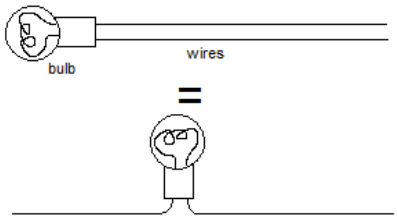
Grzzmology 101 – The study of Grzzmorgian society, with probability – Prof Janeba, Spring 2013

On the planet Woofump, the dominant species, the Grzzmorgians, have advanced to an early industrial society. In one of their cities, they are beginning to dabble in electrification, and are in the process of installing electric street lights.

A troubling flaw arises with the street light wiring; it is unwisely installed as follows:

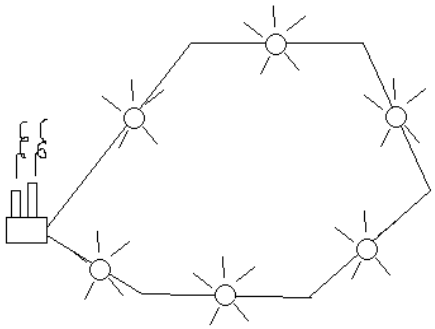
There is a central building intended as a control hub for the lighting system. Two wires come from the power station to the hub. Also, two wires from each individual street light come to the hub[1]. The power station and the bulbs are not connected to each other at all (yet). The unwise aspect of the installation is that the wires are not only unidentified, they are utterly and totally mixed up at the hub. In other words, the operators are completely unable to tell which wire is paired with another.

For each streetlight, one can think of its two wires at the hub as the two ends of a single wire, with the bulb in the middle of the wire.



Likewise, the two wire ends from the power station, which are also unidentifiably mixed in with the bulb wire ends, can be thought of as two ends of a single wire with the power station in the middle.

The design of the power station and streetlights is a high-voltage affair[2]. What this means for us is that the lights must be connected in series to the power station. In short, the Grzzmorgian light operators at the hub must connect all the wire ends in pairs (2 wire ends only in each connection) so that all the wires, including the power station wire, form one giant circle.



We don't mean that the wires are physically arranged like this, as we said, the power station's wire ends and the bulbs' wire ends all meet in the hub. That is immaterial, though – the Grzzmorgian operators have to connect the

[1] This is nuts, and doubtless says much about Grzzmorgian society, but we'll leave that for another time.

[2] Another unfortunate design flaw; it will lead to the slogan "streetlights fry Grzzmorgians," but that is not our concern in this project.

wire ends at the hub, with two wire ends in each connection, so that all the wires form a single circuit equivalent to the one drawn.

The lack of labeling on the wires is discovered on the morning of the streetlight system's grand unveiling. The chief operator didn't write the specifications for the wiring, the mess is not its[3] fault, but it has just been told to "make the necessary connections for this evening." Realizing that its superiors have put it in an impossible position, the chief says to its underlings, "OK, grzz[4] – let's just wire 'em up in random pairs and go out for lunch!" So they do – putting two wire ends in each connection, with no idea if they are making the necessary grand loop, or if they are perhaps even connecting both ends of a single bulb's wires to each other.

When the mayor turns on the power station that evening, what is the probability that the lights all go on, i.e. that the grand loop has indeed been formed?

To make this big question more tractable, we instead provide some simpler questions at first:

1. If the entire system consisted of the power station and two streetlights (3 wires total, one with a power station in the middle, two with bulbs in the middle, and a total of six unlabeled wire ends at the hub), what is the chance that the lights go on? (If this one stumps you, try 2 wires total, one station and one bulb, to get yourself started.)

2. What if we had *three* streetlights (plus a power station) – what's the probability then?

3. (This one is hard, and yes, you do have to do it): If there are three lights and a station, and X is the number of closed loops in the system's wiring after all the connections are made, give the probability distribution of X, and also give $E(X)$. Note that question 2 is asking for $P(X=1)$.

4. In fact, there were 19 streetlights. What's the probability that all the lights go on?

Do your work in groups of two or three, turn in one paper, typed, with all group member's names on it. Note that you must explain your answer in great (great!!) detail, so that you are not only right, but your paper will persuade a fellow student whose answer is *not* right. This means there will be far, far more words in your paper than equations or math symbols, though there will be plenty of those too. Because you are Willamette students, the expectation is that you will use grammatically and orthographically correct, complete sentences, seamlessly interwoven with your math symbols. See the syllabus for the (perhaps unexpected) rules about academic honesty for written group assignments. Due by Friday, Feb. 22, at 4:00 p.m. at my office.

This mini-project is worth 40 class points. It is also an opportunity for those who feel they don't or haven't tested well to raise their grades.

Group evaluation: Your group turns in one report and everyone in the group gets the same grade. To provide some accountability, however, every group member is required to submit to an informal group evaluation. Handwritten, typed, or email are OK. The evaluation should include (just!) a few sentences about how well the group worked together. The evaluation must include your estimate of the percentage of the project work that you feel each group member did (e.g. 33%/33%/33% or 40%/35%/35%, etc.). These percentages must add to 100%. These evaluations will be held in confidence, and used at the end of the semester, if necessary, to adjust final grades.

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[3] Grzzmorgians aren't genderless, it's far more complicated than that, but we don't have time to go into it here, so we will use the neuter pronoun.

[4] An informal term for coworkers on Woofump.