Further answers:

Problem: For each of the following datasets, give the mean μ and standard deviation σ. Remember that we can compute these by thinking of X as the result of a random selection from the dataset. Then μ and σ are as described in 4.3.

1. {1,2,3,4,5,6,7} $\mu = 4, \sigma = 2$
2. {0,0,0,2,2,2} $\mu = 1, \sigma = 1$
3. {5,5,5,5} $\mu = 5, \sigma = 0$
4. {1,2,3,4,5,5,6,7,12} $\mu = 5, \sigma \approx 3.055$

For each of the datasets above, using the values of μ and σ found above, verify that Chebyshev's theorem holds for k = 1,2,3 (See chapter 4.4). (Write out what Chebyshev's theorem says in each case for each of k=1,2,3 and verify that it occurs).

In set 1, Chebyshev's theorem says

k	interval	guaranteed min fraction of data in this interval	actual fraction of data in this interval
1	$(2, 6)$	$1 - \frac{1}{1^2} = 0$	$\frac{3}{7}$
2	$(0, 8)$	$1 - \frac{1}{2^2} = \frac{3}{4}$	1
3	$(-2, 10)$	$1 - \frac{1}{3^2} = \frac{8}{9}$	1

In set 2, Chebyshev's theorem says

k	interval	guaranteed min fraction of data in this interval	actual fraction of data in this interval
1	$(0, 2)$	$1 - \frac{1}{1^2} = 0$	0 (endpts not included in interval)
2	$(-1, 3)$	$1 - \frac{1}{2^2} = \frac{3}{4}$	1
3	$(-2, 4)$	$1 - \frac{1}{3^2} = \frac{8}{9}$	1

In set 4, Chebyshev's theorem says

k	interval	guaranteed min fraction of data in this interval	actual fraction of data in this interval
1	$(1.945, 8.055)$	$1 - \frac{1}{1^2} = 0$	$\frac{7}{9}$
2	$(-1.11, 11.11)$	$1 - \frac{1}{2^2} = \frac{3}{4}$	$\frac{8}{9}$
3	$(-4.165, 14.165)$	$1 - \frac{1}{3^2} = \frac{8}{9}$	1