Answers to joint probability problems:

3.42) a) $\frac{1}{20}$ b) $\frac{3}{8}$ c) $\frac{1}{2}$ d) $\frac{7}{30}$

3.46) Summing over all values of (x, y) gives the total probability of $11k$, which must equal 1, so $k = \frac{1}{11}$. Now compute $f(3, 1)$ to see the problem.

Extra problem #1: $f(x,y) = \begin{cases} kx(y-x) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$

So $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dydx = \int_0^1\int_0^2 kx(y-x)dydx = \int_0^1 \left(2kx - 2kx^2\right)dx = \frac{1}{3}k$. This must equal 1, so $k = 3$

Extra problem #2:$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(s,t)\,dtds = \int_0^x\int_0^y ks(t-s)dtds$

$= \int_0^x \left[ks\left(\frac{t^2}{2} - st\right)\right]_0^y ds = \int_0^x \left[ks\left(\frac{y^2}{2} - sy\right) - 0\right] ds$

$= \int_0^x ks\left(\frac{y^2}{2} - sy\right) ds$ (Note y is the constant here.)

$= \left.\frac{ks^2y^2}{4} - k\frac{s^3}{3}y\right|_0^x = \frac{kx^2y^2}{4} - k\frac{x^3}{3}y. = \frac{3x^2y^2}{4} - x^3y$

To check, $\frac{\partial^2}{\partial x \partial y}\left(\frac{3x^2y^2}{4} - x^3y\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{3x^2y^2}{4} - x^3y\right)\right)$

(Treat x as a constant for the derivative $\frac{\partial}{\partial y}$,) $= \frac{\partial}{\partial x}\left(\frac{3x^2y}{2} - x^3\right)$

now treat y as a constant for $\frac{\partial}{\partial x}$: $= 3xy - 3x^2 = 3x(y-x) = f(x,y)$. That checks.

3.54) $\frac{\partial^2}{\partial x \partial y}\left(\left(1 - e^{-x^2}\right)\left(1 - e^{-y^2}\right)\right) = 4xye^{-x^2-y^2}$ (for $x > 0$ and $y > 0$).

3.58) $P(a < X \le b, c < Y \le d) = F(b,d) - F(a,d) - F(b,c) + F(a,c)$