Math 266, Probability & Statistics, Spring 2013, Prof. Janeba

Answers for some chapter 8 web problems:

1. Our reading points up the oft-repeated rule of thumb, that if $n > 30$, then $\overline{X}$'s distribution may be approximated as normal, although if X's distribution is already normal, $\overline{X}$'s will be as well, for any value of n.
   Suppose that X_1, X_2, and X_3, have a uniform distribution on $[0, 1]$ (see section 6.2). Thus each X_i takes on values between 0 and 1.

   (a) What is a possible range of values for $\overline{X}$ in this context?
   Answer: averaging three values between 0 and 1 will give $\overline{X}$ between 0 and 1.

   (b) Using the information about the uniform distribution in 6.2 and what we read about Xbar in 8.2, show that the criterion in Prof. Janeba's rule of thumb, "$\overline{X}$'s distribution may be approximated as normal, if $\mu_{\overline{X}} \pm$ 2 to 3 SD's makes sense in the given context," is just barely met here. (Show the criterion is met, you need not show that the normal approximation is good; it is.)

   Answer: Using data from 6.2, X has mean $\frac{1}{2}$ and SD $\frac{1}{\sqrt{12}}$. Then $\overline{X}$ has mean $\frac{1}{2}$ also, with SD $\frac{\frac{1}{\sqrt{12}}}{\sqrt{3}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$. Thus mean $\pm$ 3 SD's for $\overline{X}$ is $\frac{1}{2} \pm \frac{3}{6}$, which stays in the realm of "sense" in this context - 0 to 1.

2. According to HANES, adult males aged 18-24 in the US have average height 70" with SD 2.5" (the values are approximate, assume for this problem they are exact).

   (a) Then for a random sample of 400 of these males, the average will be _____ (mean for Xbar) give or take _____ (SD for Xbar) or so.
   Answer: $\mu_{\overline{X}} = \mu_X = 70$", $\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{2.5}{10} = 0.25$"
   BZZT, sorry, that's wrong. For some reason I used $n = 100$ instead of $n = 400$.
   Correct Answer: $\mu_{\overline{X}} = \mu_X = 70$", $\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{2.5}{20} = 0.125$"

   (b) What is the chance that the sample average will be over 70.5"? How about 70.1"?
   CORRECTED ANSWER: We will use the normal approximation as n is quite large. $\overline{X} = 70.5$" $\rightarrow Z = \frac{70.5-70}{0.125} = 4$; $P(Z > 4) \approx 0.000\ 031\ 69$
   $\overline{X} = 70.1$" $\rightarrow Z = \frac{70.1-70}{0.125} = 0.8$; $P(Z > 0.8) \approx 0.2119$

3. (Reread problem 8.5, then do this:) A hypothetical population is 55% female and quite large. A random sample of 144 is taken (far less than 5% of the population).

   (a) The proportion in the sample that are female will be _____ (mean for Capital Thetahat), give or take _____ (SD for Capital Thetahat) or so.
   Answer: $\mu_{\hat{\Theta}} = \theta = 0.55$, $\sigma_{\hat{\Theta}} = \frac{\sqrt{\theta(1-\theta)}}{\sqrt{n}} = \frac{.4975}{12} = 0.04146$

   (b) What is the chance that the sample proportion will be under 50%?
   Answer: $\hat{\Theta} = 0.50 \Longrightarrow Z = \frac{0.50-0.55}{0.04146} = -1.21$; $P(Z < -1.21) \approx 0.1131$